

**Ecole Nationale Supérieure
de l'Aéronautique et de l'Espace**

**SPACE MECHANICS
Study Report**

***Subject : Influence of the Surface Forces on the Orbit
of the SPOT Satellite.***

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Surface Forces Influence on an Earth Observation Satellite like SPOT

The purpose of this project is to study the surface forces' influence on an Earth Observation Satellite. Indeed, 'orbit determination' enables one to study the effect of these surface forces that were usually neglected. In this project we try to model as precisely as possible the satellite and the surfaces forces on it, so that we have a very accurate idea of the position of the satellite. For a Earth observation satellite, this is a very important parameter. *an*

To accurately compute the surfaces forces, one must first model the satellites dimensions, surfaces, optical properties and drag coefficients. Then one must compute the surface forces, and then after it's possible to propagate the orbit.

The satellite model was given by the CNES (the French space agency). Indeed, the satellite is SPOT 2. Two different software were used to compute the thermal forces and to propagate the orbit. Respectively :

UTOPIA : University of Texas Orbit Propagator

TRASYS : a Martin Marietta Code

For our project we have developed an accurate drag coefficient model, and modified the Utopia Code so that the model can fit in it.

With all this background, we have been able to study the effect of the following perturbations on the orbit : J2, atmospheric drag, and thermal forces. For each of these perturbations we try to develop a simple mathematical model, to better understand their influence. For the atmospheric drag perturbation, we have used different atmospheric density models for comparison.

The following paragraphs will at first present the SPOT satellite in general, and then more precisely the study.

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An Earth Observation Satellite : SPOT

The SPOT Satellite [Ref. 1] .

SPOT is the French Earth-observation satellite. Two models have already been launched by the European rocket ARIANE : SPOT-1 (launched in 1986) and SPOT-2 (launched in January 1990) which carries the very accurate tracking system DORIS. Two new versions of SPOT are planned to be launched in the years to come.

The SPOT mission is characterized by a high spatial resolution in the visible : 10 m in panchromatic mode (large bandwidth) , 20 m in multispectral mode (three bandwidths in visible or close infra-red). Made of 60x60 km, the SPOT images have multiple applications such as :

- # mapping, urban planning, land resources management.
- # geology, mineral and oil research.
- # agriculture, terrain evaluation, forestry.
- # coastal zone studies.

Sun-synchronized, the SPOT orbit provides a constant illumination of the observed zones and the repeatability of the observations. Thus, from an altitude of 820 km, the satellite will pass over the same point every 26 days, after 369 orbits. A steerable mirror located at the input of the two instruments enables imaging to be performed at nadir or offnadir viewing : this capability offers several advantages unique to SPOT : access to a preselected zone, repeatability of the observations for a given zone, and stereoscopic observation.

The SPOT satellite is of modular design and consists of two assemblies :

- # the Multimission Platform, will be re-used by other observation program such as ERS (Earth Remote Sensing satellite from ESA).
- # the payload specific to the SPOT mission consisting of two high resolution visible instruments.

The multimission platform :

The multimission platform developed by MATRA, a French company, for the SPOT program has been designed in order to be re-used easily by other low orbiting satellites. Thus, the ERS-1 program of ESA will use the maximum version of the platform. Optimized for Earth observation, the platform is characterized by accurate and very stable geocentric pointing, an important instantaneous electrical power, particularly required by microwave instruments, and a large functioning autonomy. By design, the Solar Array size, the number of batteries, and the propellant mass can be adopted for a given mission. For the same purpose the overall satellite management is automatically executed by a specific software loaded in the on-board platform memories.

The High Resolution Visible Camera :

Two model of this camera constitute the payload of SPOT spacecraft. The focal plane of that camera is equipped with 4 linear arrays of 6000 sensors, electronically scanned, the spacecraft orbital motion producing the other scanning direction. Analog signals provided by the detectors are amplified and digitized by electronics. Integrated to high resolution visible pictures, then sent to the payload telemetry subsystem which ensures data transmission to the ground. A flat mirror is located at the entrance of the instrument allowing side viewing 400 km on each side of the sub satellite track. A specific position of this mirror ensures an in-flight calibration using halogen lamps, or solar flux picked up by optical fibers.

Earth Observation Constraints on the SPOT Orbit [Ref. 2]

An Earth observation satellite must have the following properties :

Quality of the pictures :

The quality is a function of the altitude of the satellite. High altitude means a good mapping of the Earth, but a low resolution. Low altitude means high resolution, poor mapping, and high atmospheric drag.

Moreover, the pictures will be assembled to draw maps, and will be compared to each others. Thus, it is very important that they all have the same scale, that means the satellite has to remain at the same altitude on its orbit or have a very small eccentricity.

These quality constraints determine to choose a quasi-circular orbit with a very small eccentricity less than 2×10^{-3} , and an altitude about 800 km.

Constant light :

So that the shots have a certain unity, it is necessary that the light condition is similar on each picture. Furthermore cameras are very sensitive to the solar light reflected by the Earth, which varies with the solar time. Therefore, the stability in time of the solar hour gets rid of parasite phenomena and day-evolution which spoil long-term Earth study.

This necessity of constant light is satisfied by a sun-synchronous orbit. For a circular orbit we have to choose the semi-major axis knowing the inclination. Indeed, the sun-synchronism is made possible by the oblateness of the Earth. In the development of the Geopotential Energy the J_2 -term represents this oblateness.

Then we can write the Earth-potential as :

$$V = \left(1 - \frac{1}{2} J_2 \left(\frac{R_t}{r} \right)^2 \cdot (3 \sin^2 \varphi - 1) \right)$$

where r is the distance satellite-Earth center
 φ is the latitude of the satellite
 R_t is the Earth radius
 J_2 is the second zonal harmonic

If we write the Lagrange equation for $d\Omega/dt$ with this Earth-potential it comes out :

$$\frac{d\Omega}{dt} = -\frac{3}{2} n \left(\frac{R_t}{a} \right)^2 \cdot J_2 \cdot \frac{\cos i}{(1 - e^2)^2}$$

Furthermore, the Earth-orbit has a period of 365.25 days, then the mean daily rotation of the Sun around the Earth is :

$$\frac{d\alpha}{dt}^m = \frac{365}{365.25} = 0.98562^\circ / \text{day}$$

If we take the orbit parameter so that $d\Omega/dt$ equals to this rotation, the nodes of the orbit will remain at the same solar hour in average. Assuming eccentricity, $e=0$, the sun-synchronous relation between semi-major axis, a , and inclination, i , becomes :

$$\cos i = -\frac{0.985}{9.97} \cdot \left(\frac{R_t}{a} \right)^{-\frac{7}{2}}$$

The angle between the node line and the sun meridian, θ , remains constant without regards to the fluctuations of the time equation (changes in the Earth rotation velocity around the Sun due to the ellipticity of the Earth orbit) (see figure next page).

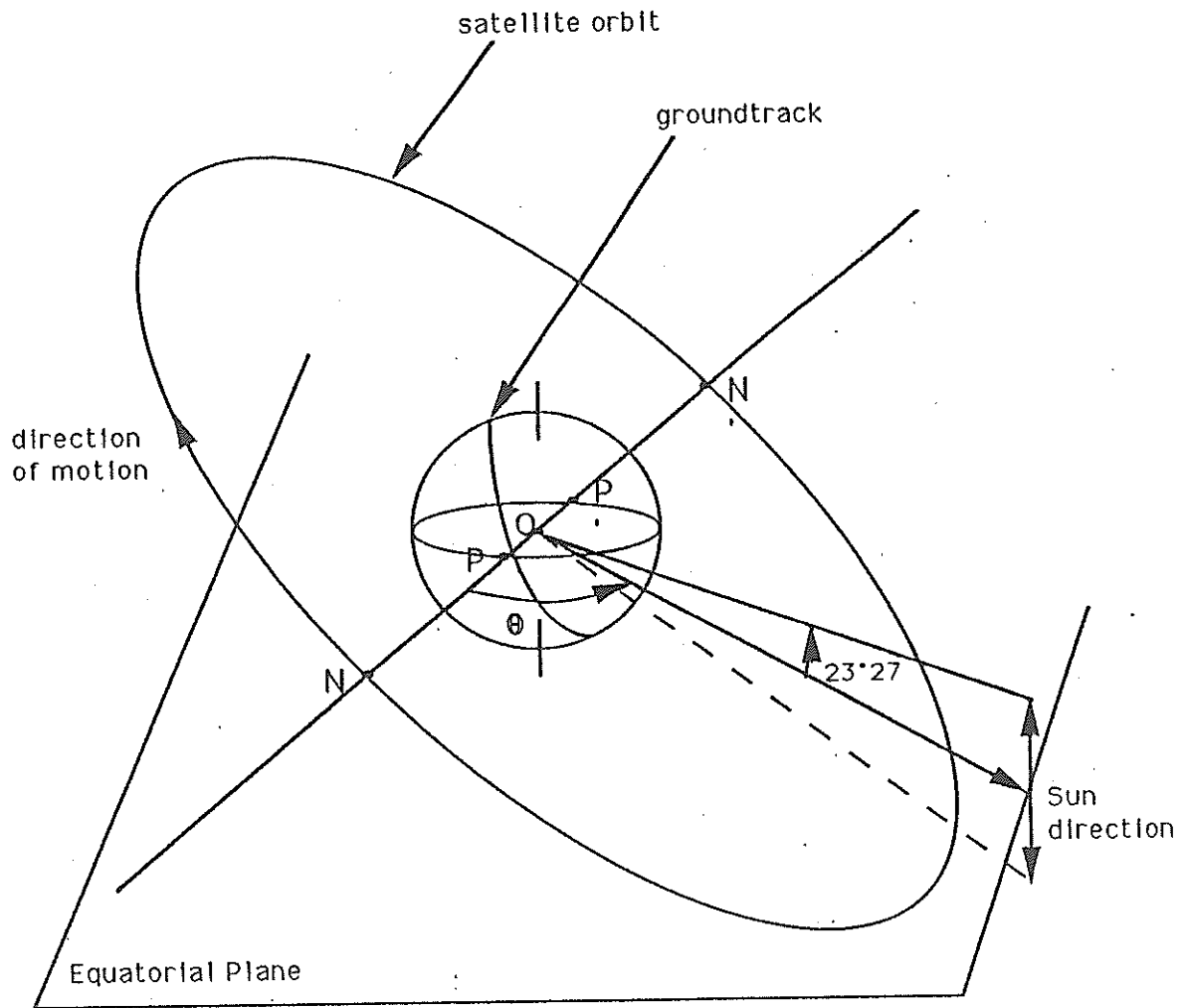
Let α_{sm} be the mean visible motion of the Sun, α_s the true visible motion, and H the local time of ascending node, then we have the following equation :

$$H = 12h + \Omega - \alpha_s = 12h - \theta$$

$$H = 12h + \Omega - \alpha_{sm} + TE = H_{\text{mean}} + TE$$

The change in TE (time equation) can reach 16 minutes. The local time remains constant in average. Along the time, this change induces some variation in the amount of light during the orbit for a similar latitude. Moreover because of the obliquity (modification of the ecliptic inclination on the equator) the light changes also along the season.

SUN- SYNCHRONIZED SATELLITE : $\theta = \text{Constant}$



Repeatability of the pictures and mapping of the Earth :

To study a part of the Earth and look at its variation with respect to time, it's easier when the satellite flies over the same point periodically. Moreover, the groundtrack of the satellite should cover as much of the Earth's surface as possible .

This influences one to choose an orbit in phase with the Earth. To map the Earth with accuracy, the mapping-period has to be of a couple of days. In order to make the repeatability possible, the number of orbit revolutions per day has to be a rational number.

Within a day the satellite accomplished p orbit, with :

$$p = n + m/q$$

where $m < q$

m and q integers which have no common factors

Within q days, the satellite has accomplished $(q.n + m)$ revolutions, and the first track of the day $(q + 1)$ will superimpose the first track of the first day.

The three integers m, n and q were chosen so that the cover of the Earth is as complete as possible. If Δ is the distance between two groundtracks, Δ must be smaller than $(1 - r) C$

where r is the cover ratio

C is the field of a picture

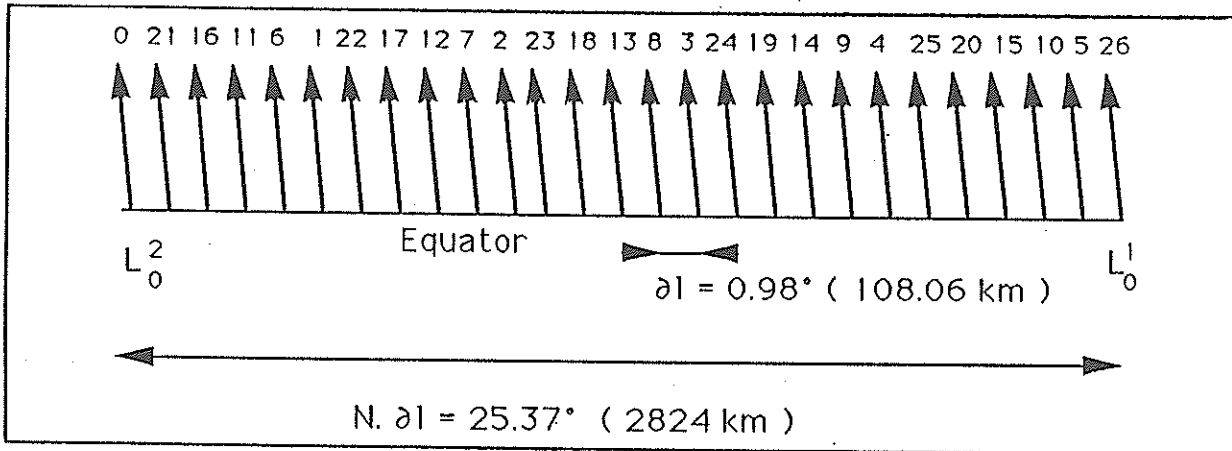
Where R_t is the Earth Radius.

The gap between two successive tracks is $2\pi/p$. The set of Ascending nodes' West Longitude of the first orbit of the j^{th} day is computed in the following way : on the j^{th} day, the groundtrack move west of $(1 - m/q) \cdot 2\pi/p$ from the tracks of the $(j - 1)^{\text{th}}$ day. Then we can write the West Longitude of ascending nodes of the first orbit of the j^{th} day :

$$L_j^1 = L_0^1 + \frac{2\pi}{p} \left[j \cdot \left(1 - \frac{m}{q} \right) \pmod{q} \right]$$

or

$$L_j^1 = L_0^1 + \frac{2\pi}{p \cdot q} \left[q - m \cdot j \pmod{q} \right]$$



Ascending nodes of SPOT along the Equator

The choice of the orbit number per day, p , is linked to the orbital period, and consequently to the altitude :

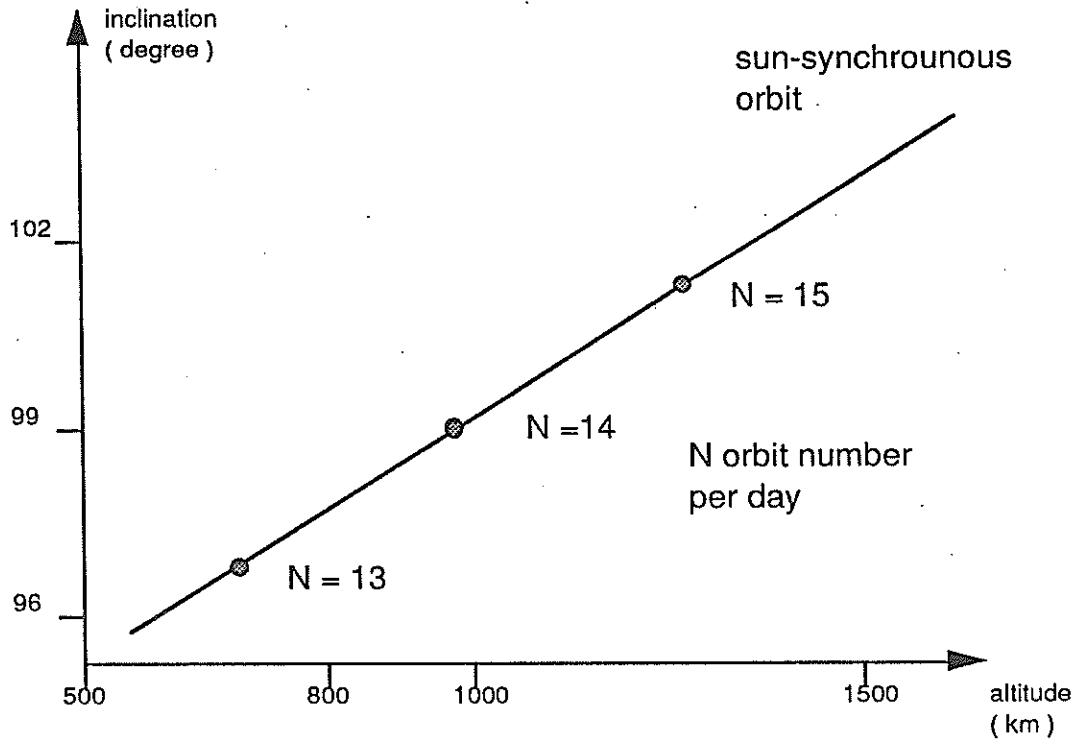
$$T = \frac{2\pi}{\frac{d\omega}{dt} + n} = \frac{2\pi}{\frac{d\omega}{dt} + \sqrt{\frac{\mu}{a^3}}}$$

In a first estimation we can disregard $d\omega/dt$ in front of n (in the case of SPOT, they try to keep $d\omega/dt$ small, and ω constant to avoid altitude fluctuations), then :

$$T \approx 2\pi \cdot \sqrt{\frac{a^3}{\mu}}$$

The choice of the semi-major axis, a , is also linked to mapping problem, and the choice of the inclination, i , (sun-synchronized condition). The figure show the relation between altitude, inclination and orbit numbers per day.

Sun-synchronous constraint between the altitude and the inclination



Therefore, SPOT parameters are :

$h \approx 820 \text{ km}$
 $n = 14$
 $m = 5$
 $q = 26$

This implies a 26-day orbit cycle, a 369-orbit grid, a distance between two groundtracks of 108 km on the Equator, and of 76 km at 45° Latitude. And the field of the two instruments is 117 km.

The distance between two consecutive tracks on the Equator is 2824 km, and the choice of repeatability, allows to obtain for a period shorter than 26-days a quasi-repeatability for the orbit. Indeed, if an integer r smaller than q is so that :

$$r.p = r.n + \frac{r.m}{q} = \text{integer} \pm \frac{1}{p}$$

After r days, the satellite will have performed a number of quasi integer revolutions. The ascending nodes for the first orbit of the $(r+1)^{\text{th}}$ day will be at :

$(\pm \frac{2.\pi}{p})$ from the ascending node of the first orbit of the first day. The parameter, r , is a sub-cycle.

Choice of the Orbit Parameters

Choice of the argument of the ascending node, Ω :

We have just seen the reason of a sun-synchronous orbit for Spot. We have now to choose the right local hour, which will remain almost constant along the time.

Three constraints drives to choose Ω :

- have the best light as possible for the shots : 12:00 am, 12:00 pm orbit.
- reduce the part of the orbit in the shadow, so that the solar array is as much as possible in the sunlight, and assure then a good energy power : ideal for a 6:00 am, 6:00 pm orbit.
- avoid the phenomena of specularity, which appears when the angle sun-view point, satellite-view point is small. This effect is maximal when the local hour is around 12:00.

Knowing these constraints, the local hour must be chosen in the light side from 10:00 to 11:00 or from 13:00 to 14:00.

The choice of 10:30 has been made, considering the observable surfaces between the North and South Hemisphere, when the satellite is in an ascending orbit or descending orbit.

A descending-orbit having a larger observable surface in the North Hemisphere, that is this one which has been chosen : $\Omega \approx 10:30$ pm.

Choice of the argument of the perigee, ω , and the eccentricity, e :

The altitude constant constraint drives to choose a small eccentricity. But because of the oblateness of the Earth, if the orbit were circular ($e=0$) the altitude would be higher around the poles. The best is to choose e non equal to zero and mean ω around the pole :

$$e \approx 1.1 \cdot 10^{-3}$$
$$\omega \approx 93^\circ$$

Choice of the semi-major axis, a , and the inclination, i :

The sun-synchronous orbit gives a relation between a and i . And the choice of a implies the mapping of the Earth. Therefore one has chosen :

$$i = 98.723^\circ$$
$$a \approx 7200.5 \text{ km}$$

Orbital Characteristics chosen for the study

For all this study, one have chosen the following parameters, they are not exactly the same as the one suggested in the previous paragraphs, but correspond to the osculating orbit of SPOT on June 23, 1989.

Kepler parameters on June 23,1989 at 5°0'24" U.T. :

$a = 7205 \text{ km}$
 $e = 1.51 \cdot 10^{-3}$
 $i = 98.7^\circ$
 $\omega = 102.5^\circ$
 $\Omega = 249.7^\circ$
 $M = 287.0^\circ$

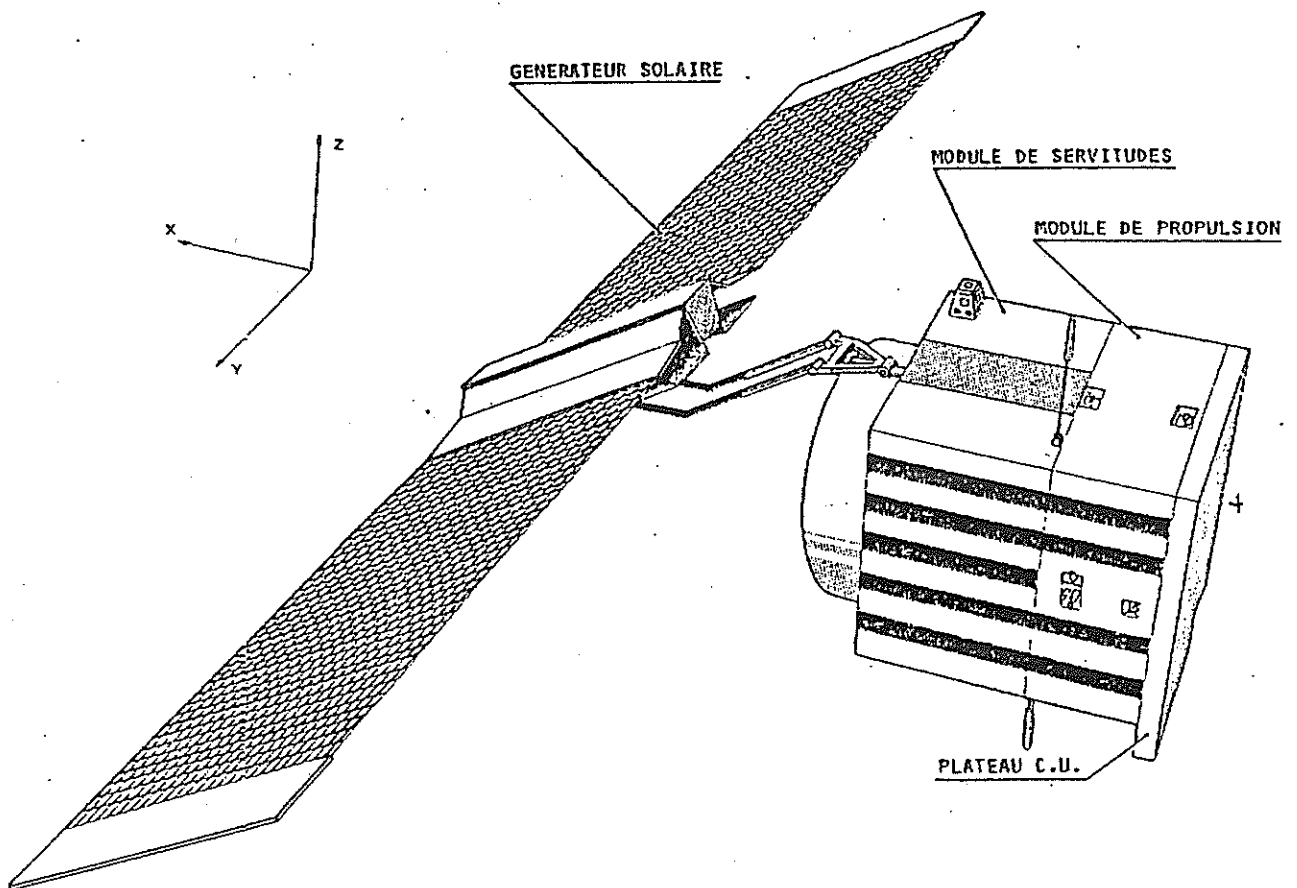
Other Orbital characteristics :

orbital period : 6086 sec (i.e. 101 min 36 sec)
altitude : 827 km
periodicity : 26 days
mass : 1850 kg

Geometric Model of SPOT

The following picture is a 3D view of SPOT, or more precisely of the multimission module and the solar array (the payload, which are for SPOT cameras, is not shown on this picture).

Appendix A.1 and A.2 represent the trapezoidal model of SPOT that we have used to compute the surfaces forces.



CONFIGURATION ORBITALE DE LA PLATE-FORME MULTIMISSION

Perturbation Forces Computation

Introduction

The purpose of this paragraph is to compute the perturbation forces on the SPOT satellite. The perturbation forces taken into account are computed either within the program Utopia (University of Texas orbit propagator) or with the program TRASYS (Martin Marietta code) or using other independent programs depending on the nature of the perturbations :

Perturbation forces computed within the program Utopia :

- * Non-spherical attraction of the earth,
- * Moon & sun attraction,
- * Relativistic effects,
- * Atmospheric drag (external model that we developed for the purpose of this study).

Perturbation forces computed external to Utopia :

- using the program TRASYS (Martin Marietta)
 - * Direct solar irradiation
 - * Albedo flux
 - * Terrestrial infrared flux
- using the programs developed for this study :
 - * program SATIR for the infrared emission of the satellite
 - * program DRAG computing the drag and lift coefficients that are used as an external input for the atmospheric drag perturbation.

Computation of the thermal forces using the program TRASYS (Thermal Radiation Analysis SYStem).

Presentation

TRASYS is a program developed at Martin Marietta which calculates the radiation fluxes from solar, albedo and Earth IR incident upon each surface of the spacecraft. These fluxes are then converted to resultant forces based on the surface properties and orientation of each surface.

The inputs of TRASYS are :

- Satellite geometry using rectangles, trapezoids, discs
- Orientation model
- Surface optical properties

Assumptions

Geometrical model (See appendix A)

The geometrical model adopted is composed of :

- a rectangle for the solar array (4 nodes)
- a trapezoidal box for the body of SPOT (7 nodes)

Actually, this model is close to reality since the shape of SPOT is fairly simple.

Orientation of SPOT on its orbit

Surface S1⁻ of the main body is directed toward the Earth at all times and surface S2⁺ is almost always normal to the velocity vector (not exactly because of the small eccentricity of the orbit).

Solar array orientation

The solar array must always be normal to the sun vector. Since SPOT is a sun-synchronous satellite (the plane containing the orbit is fixed relative to the direction of the sun), meeting this requirement is relatively easy. Indeed, a single rotation of the solar array is enough to insure the normality at all times.

Optical properties

The optical properties have been determined for each node with a distinction being made between the visible coefficients and the infrared coefficients. Assuming that the transmissivity is zero, these optical properties are the absorptivity (α or ϵ), the specular reflectivity component (KS^v or KS^i) and the diffuse reflectivity component (KD^v or KD^i).

$$\text{Visible} \quad \alpha + KS^v + KD^v = 1.0$$

$$\text{IR} \quad \epsilon + KS^i + KD^i = 1.0$$

The values of the coefficients used in the calculation are issued by CNES and are listed in appendix B. Sometimes, the symbol β is used to refer to the percentage of reflected radiation which reflects in a specular manner.

$$KS^v = \beta (KS^v + KD^v)$$

$$KS^i = \beta (KS^i + KD^i)$$

Solar constant [Ref. 3]

The total emissive power of the sun is $E_{\text{sun}} = 3.826 \times 10^{26}$ W. The flux of this energy G_{sun} varies inversely proportional to the square of the radial distance r from the sun. Intercepted by Earth, at a mean distance of 1 A.U., this constant time rate of flow of radiant energy per unit area is known as the solar constant or solar radiation flux density. Since earth's orbit is eccentric, the solar radiation flux received by the earth system varies slightly throughout its orbit around the sun. Therefore, the solar flux is computed as a function of the Earth - Sun distance which is deduced from the Julian date.

$$M = 2\pi(\text{JD} - \text{JD}(\text{Januar 1, 89, 0h}))/365.25$$

$$v = M + (2e - e^3/4)\sin M + (5e^2/4)\sin 2M + (13e^3/12)\sin 3M$$

$$r = a(1 - e^2)/(1 + e \cos v)$$

$$G_{\text{sun}} = 3.826 \times 10^{26}/4\pi r^2$$

with :	M	mean anomaly (rad)
	JD	Julian date (day)
	v	true anomaly (rad)
	e	Earth eccentricity (e=0.0167)
	a	Earth mean distance (a=1.496 10 ¹¹ m)
	r	Earth - Sun distance (m)
	G _{sun}	solar radiation (Wm ⁻²)

For June 23rd, the solar radiation is 1317 Wm⁻².

Albedo and emissivity of the Earth [Ref. 3]

Albedo and emissivity of Earth can be represented as a spherical harmonic expansion. In our case, a 2 degree zonal harmonic model is chosen which takes into account variations with the season and the latitude (See appendix C).

$$a = a_0 + a_1 P_1(\sin \phi) + a_2 P_2(\sin \phi)$$

$$\varepsilon = \varepsilon_0 + \varepsilon_1 P_1(\sin \phi) + \varepsilon_2 P_2(\sin \phi)$$

where ϕ is the latitude and P_1 and P_2 are the first and second degree Legendre polynomials. The coefficients a_0 , a_2 , ε_0 , ε_2 are constants and the first degree coefficients a_1 and ε_1 have a seasonal variation,

$$a_1 = c_0 + c_1 \cos(\omega(t-t_0)) + c_2 \sin(\omega(t-t_0))$$

$$\varepsilon_1 = k_0 + k_1 \cos(\omega(t-t_0)) + k_2 \sin(\omega(t-t_0))$$

respectively, t is the time in question, t_0 is the epoch of periodic terms, ω the orbital period of the earth, and c_0 , k_0 , c_1 , k_1 , c_2 , k_2 are constant parameters [ref. 3].

Theory [Ref. 3]

Knowing these assumptions about the fluxes, the resulting forces are calculated using the following formula.

$$F_i = - \frac{G_i A_i \cos \theta_{ij}}{c} \left((1-\gamma_i \beta_i) s_j + (D_i(f) \gamma_i (1-\beta_i) + 2\gamma_i \beta_i \cos \theta_{ij}) n_i \right)$$

This formula allows us to compute the force exerted on the satellite surface i due to the incident radiation from source j . The meaning of the symbols used are :

- F_i : Radiation force on surface i
- G_j : Incident radiation from source j per unit of area
- A_i : Area of satellite surface i
- s_j : Unit vector from surface i toward source j
- n_i : Unit vector normal to surface i
- θ_{ij} : Angle subtended by s_j and n_i
- c : Speed of light
- $D_i(f)$: Diffusion function for surface i
- γ_i : Reflectivity coefficient for surface i
- β_i : Specular reflectivity coefficient for surface i

We consider a Lambert diffusion ($f = \cos \theta$), then $D(f) = 2/3$

Depending on the source, G_j is calculated using the following formulas :

$$j = \text{sun} \quad G_j = \frac{3.826 \cdot 10^{26}}{4 \pi r_j^2}$$

$$j = \text{earth albedo} \quad G_j = \frac{a_i G_{\text{sun}} \cos \theta_{si}}{\pi r_j^2} \cos \theta_{s/c} dA_j$$

$$j = \text{earth IR} \quad G_j = \frac{\epsilon_i G_{\text{sun}}}{4 \pi r_j^2} \cos \theta_{s/c} dA_j$$

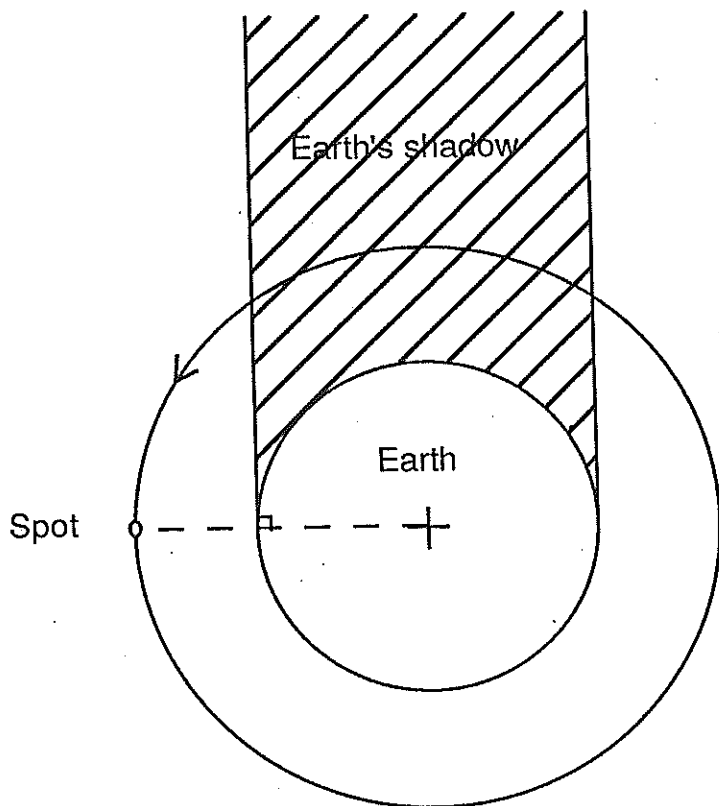
A drawing explaining some of the symbols is given in Appendix D.

Results

The accelerations resulting from the solar radiation, the albedo and the infrared emission of the Earth have been computed along one orbit and are given in Appendix E. The origin of the orbit angle has been defined as indicated below (sunrise on the earth).

The solar force appears to be the most important acceleration compared to planetary and albedo accelerations. Moreover, the component due to the solar array is the main part of

the solar force because of its area and position relative to the sun (always perpendicular). The computation of the solar force has been checked in a sample case (solar array only) by comparing the computed results from TRASYS and the theoretical results (see appendix H).



Position of the Spot satellite for an orbit angle of 0°

The infrared emission of the satellite

To compute this force we have written the program SATIR which computes the infrared forces knowing the temperature of each surfaces, their directions and their optical properties (See appendix F).

Assumptions

The solar array emission is not taken into account because the temperatures of the two sides of this array are supposed to be equal (Experimentally, the temperature gradient in the array is smaller than 1K).

The main body of the satellite has been divided into 12 nodes according to the different materials of each face. All of the node characteristics used in the computation like area, emissivity, normal vector and temperature are given in appendix G.

The temperature of each node have been chosen using experimental data :

- First of all, all the faces covered with kapton have with a good precision a constant temperature in the shade (188°K) and in the sun (338°K) except for the face 5 which always faces the earth (this face is always at 273°K).

- Moreover, the faces covered with SSM have a sinusoidal variation of their temperature with an amplitude depending on the face considered. For face 10, the solar flux conditions are constant out of the Earth's shadow and therefore its temperature is always 275°K.

Theory [Ref. 3]

The force resulting from the infrared emission of the satellite is calculated by applying the following formula to each node.

$$F_i = - \frac{D(f) \epsilon_i A_i \sigma T_i^4}{c} n_i$$

F_i : Radiation force on surface i

A_i : Area of satellite surface i

n_i : Unit vector normal to surface i

c : speed of light

$D(f)$: Diffusion function

ϵ_i : emissivity of surface i

σ : Stefan-Boltzmann constant ($5.669 * 10^{-8} \text{ Wm}^{-2}\text{K}^{-4}$)

We consider a Lambert diffusion ($f = \cos \theta$), $D(f) = 2/3$
The total force is then calculated with respect to the orientation of each face.

Results

The acceleration resulting from the infrared emission of the satellite has been computed along one orbit and is given in Appendix I. The magnitude of this acceleration is of the order of 2 nms^{-2} and therefore very small compared to the other accelerations.

The atmospheric drag

It's possible to compute the atmospheric drag within Utopia using a constant drag coefficient along one orbit. But, because of the motion of the solar array, the aerodynamic coefficients change along one orbit. Therefore, in order to be as accurate as possible, the products of the area by the aerodynamic coefficients of the satellite (SC_t , SC_n , SC_r) have been computed along one orbit by the program DRAG, that we have written (See Appendix J). Then, after some changes in some subroutines of the Utopia code, this data is used as inputs in the program Utopia to calculate the atmospheric drag. These calculations require atmospheric data (densities) that are deduced from atmospheric models. Three of these models are available within Utopia (Jacchia 71, Jacchia 77, DTM) and can be selected by the user.

The assumptions [Ref. 4]

For the computation of the aerodynamic coefficients, we have made the assumption of 'free molecular flux' with a diffuse reemission (Cook's assumption).

The inputs of the program DRAG are :

- area & temperature of each node (See appendix K)
- mean molecular weight of the incident molecules

The mean molecular weight, WMM, is computed by the program MASMOY (See appendix L) using this formula :

$$WMM = \frac{2 \rho(H_2) + 4 \rho(He) + 16 \rho(O) + 28 \rho(N_2) + 32 \rho(O_2)}{\rho(H_2) + \rho(He) + \rho(O) + \rho(N_2) + \rho(O_2)}$$

with H₂, He, O, N₂ and O₂ being the main components of the atmosphere along the orbit of Spot. The data of densities for these components are calculated using one of the atmospheric model included in Utopia. The variation of densities temperature and mean molecular weight along one orbit are presented in appendix M.

- Velocity incidence angle $\theta(i)$

$\theta(i)$ is defined for each surface as the angle between the velocity vector of the surface and the surface itself (a drawing explaining the geometrical meaning of θ and the formula for the calculation of θ are given in appendix N).

The theory [Ref. 4]

The aerodynamic coefficients have been calculated separately for each node. In each case, the lift and drag coefficients due to the absorption and reemission have been computed separately using the following formulas .

C_{Da} : drag coefficient due to the absorption

$$C_{Da} = \sin\theta \left(1 + \frac{1}{2s^2}\right) (1 + \text{erf}(s \sin\theta)) + \frac{1}{s\sqrt{\pi}} \exp(-(s \sin\theta)^2)$$

C_{Dr} : drag coefficient due to the reemission

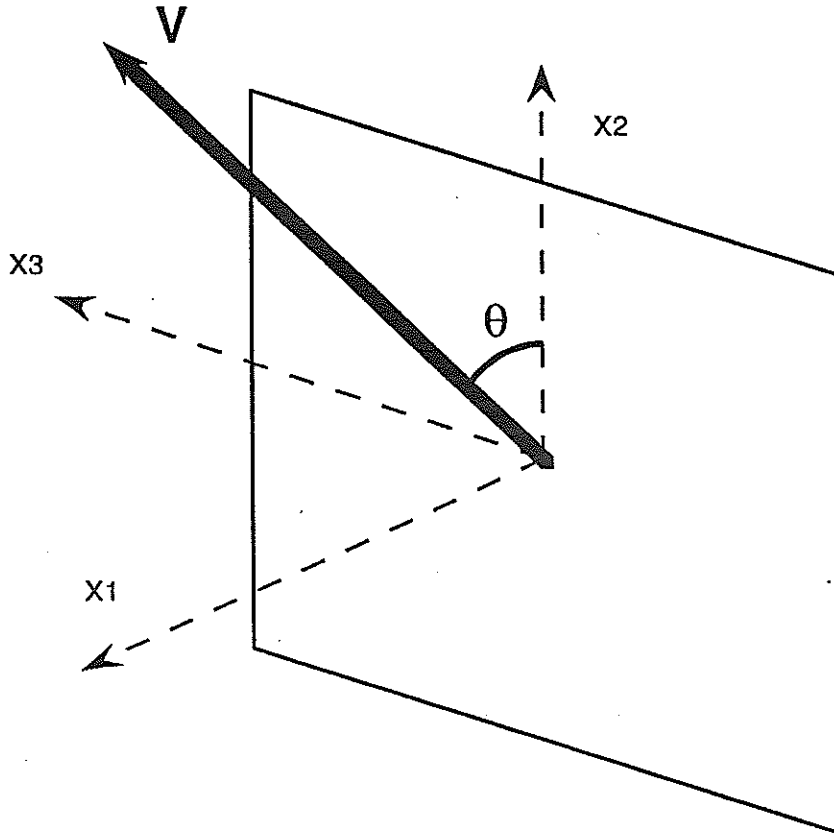
$$C_{Dr} = \frac{\sqrt{\pi} \sin^2\theta}{2s} \sqrt{\frac{T_w}{T}} (1 + \text{erf}(s \sin\theta)) + \frac{\sin\theta}{2s^2} \sqrt{\frac{T_w}{T}} \exp(-(s \sin\theta)^2)$$

C_{La} : lift coefficient due to the absorption

$$C_{La} = \frac{\cos\theta}{2s^2} (1 + \text{erf}(s \sin\theta))$$

C_{Lr} : lift coefficient due to the reemission

$$C_{Lr} = \frac{\sqrt{\pi} \cos\theta \sin\theta}{2s} \sqrt{\frac{T_w}{T}} (1 + \operatorname{erf}(s \sin\theta)) + \frac{\cos\theta}{2s^2} \sqrt{\frac{T_w}{T}} \exp(-(s \sin\theta)^2)$$



$X1$: Unit vector normal to the surface

$X2$: Unit vector located on the surface and the vector V is in the plan ($X1, X2$)

With :

$$\sqrt{T_{wf}} = \sqrt{T_p} + \sqrt{1-\alpha} \left(\sqrt{\frac{V^2}{2R}} - \sqrt{T_p} \right)$$

$$\sqrt{T_{wr}} = \sqrt{T_p} + \sqrt{1-\alpha} \left(\sqrt{\frac{V^2}{2R}} - \sqrt{T_p} \right)$$

$$s = V/\sqrt{2RT}$$

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp-z^2 dz$$

and :

θ	Velocity incidence angle of the face (rad)
T	Absolute temperature of the atmosphere (K)
V	Velocity of the satellite (ms^{-1})
R	Boltzmann's gas constant ($\text{kgm}^2\text{mole}^{-1}\text{s}^{-2}\text{K}^{-1}$)
T_p	Face temperature (K)
T_w	Face gas temperature (K)
T_{wf}	in the case of a front face
T_{wr}	in the case of a rear face

As the faces $S1^+$, $S1^-$, $S3^+$, $S3^-$ (see notation in appendix A) are always parallel to the velocity, their lift and drag coefficients is are nearly zero. Moreover, the face $S2^-$ is at the rear part of the satellite and then, because of the high velocity of the spacecraft, the lift and drag coefficients are equal to 0. On the other hand, the face $S2^+$ is always normal to the velocity. Therefore, the drag coefficient is important (See appendix O) but the lift coefficient is always equal to zero.

For the solar array, the coefficients change with time because of its own motion (When the node is at the back, the coefficients are equal to 0 - See appendix P).

The total SC factors are then calculated using the following formulas.

$$SC_t = \sum_{i=1}^{14} A^{(i)} (C_{Da}^{(i)} + C_{Dr}^{(i)})$$

$$SC_n = \sum_{i=1}^{14} A^{(i)} (C_{La}^{(i)} + C_{Lr}^{(i)}) \cos\tau$$

$$SC_r = \sum_{i=1}^{14} A^{(i)} (C_{La}^{(i)} + C_{Lr}^{(i)}) \sin\tau$$

where τ is the angle between the lift vector and the normal axis. A drawing is given in appendix Q to explain the meaning of τ . Moreover, the formulas used to calculate the values of $\cos \tau$ and $\sin \tau$ are given and demonstrated in the same appendix.

Results

The products SCt, SCn, SCr have been computed along one orbit and are given in Appendix R. The force due to the atmospheric drag is computed by UTOPIA within the following formula :

$$F_d = \frac{1}{2} \rho SC_d V^2$$

This force computed along one orbit is given in appendix S.1. The transversal solar force has a very characteristic twice per revolution signature due to the motion of the solar array. This force ranging from 10 to 35 nms⁻² is composed of the drag of the body of the satellite (almost constant component) and the drag of the solar array (almost sinusoidal component).

Orbit Propagation

Introduction

The orbit propagator from the University of Texas at Austin named UTOPIA has been used. The perturbation forces taken into account are specified in the input file of UTOPIA (see appendix ...). For SPOT, the forces used for the calculation in UTOPIA follows :

- the Earth attraction represented by a spherical harmonic expansion (36 x 36 GEMT2 model)
- the atmospheric drag which is calculated within UTOPIA , the SC factor (the product of the surface by the aerodynamic coefficient) being entered through the file CDRAG. For this calculation, it's possible to choose on the one hand one of the 3 atmospheric model available within UTOPIA and on the other hand to enter any geo-magnetic flux data.
- the thermal forces due to the direct solar radiation, the albedo of the Earth, the IR radiation of the Earth and the IR radiation of the satellite itself are entered in UTOPIA through the file RPACC.
- the relativistic perturbation
- the perturbation due to the dynamic solid earth tide
- the perturbation due to the general relativity
- the perturbation due to the polar motion
- the perturbation due to the lunar tides
- the perturbation due to the solar attraction
- the perturbation due to the lunar attraction

All these forces have been printed and are given in appendix P.

Evolution of the orbital parameters along one orbit

Presentation of the results

The 6 orbital parameters released by UTOPIA are a , e , i , ω , Ω , $\omega+f$. The program UTOPIA has been executed for a one orbit arc with the conditions described in the first paragraph. The variations of the 6 orbital parameters along one orbit are outputs of the program and are given in appendix U.

The variations of the orbital parameters seem to follow the equations written below :

$$a(\text{km}) = 7200 + 9.5 \cos(2\alpha + \phi_1)$$

e_x : sinusoidal variation with harmonics α and 3α

e_y : sinusoidal variation with harmonics α and 3α

$$i(\text{deg}) = 98.763 - 5.5 \cdot 10^{-3} \cos(2\alpha + \phi_2)$$

$$\Omega(\text{deg}) = 249.82 + 1.1 \cdot 10^{-2} \alpha + 0.005 \sin(2\alpha + \phi_3)$$

These can probably be demonstrated to be driven mostly by J_2 .

with α the mean longitude of the satellite.

Theoretical calculations

Introduction

A very simple and rapid method is developed to study the effect of the J_2 -term perturbation on the low & circular orbit of a satellite. For that purpose, we will use the Gauss equations established for a circular orbit [Ref. 4].

The following results will explain the shape in '8' of the eccentricity vector.

Method [Ref. 5]

The idea is to do a development by neglecting the first order of the eccentricity in the Gauss equations. That means to make the following assumptions :

$$r \approx a$$

$$e \approx 0$$

$$V \approx a \cdot n = \text{cst}$$

$$\alpha_v = \omega + v \approx \alpha = \omega + M$$

That gives the Gauss system, for a circular orbit (given the partials of the parameters knowing the perturbation accelerations)

$$\frac{da}{dt} = 2.a.\frac{T}{V}$$

$$\frac{de_x}{dt} = \frac{2.T}{V} \cdot \cos \alpha + \frac{R}{V} \cdot \sin \alpha$$

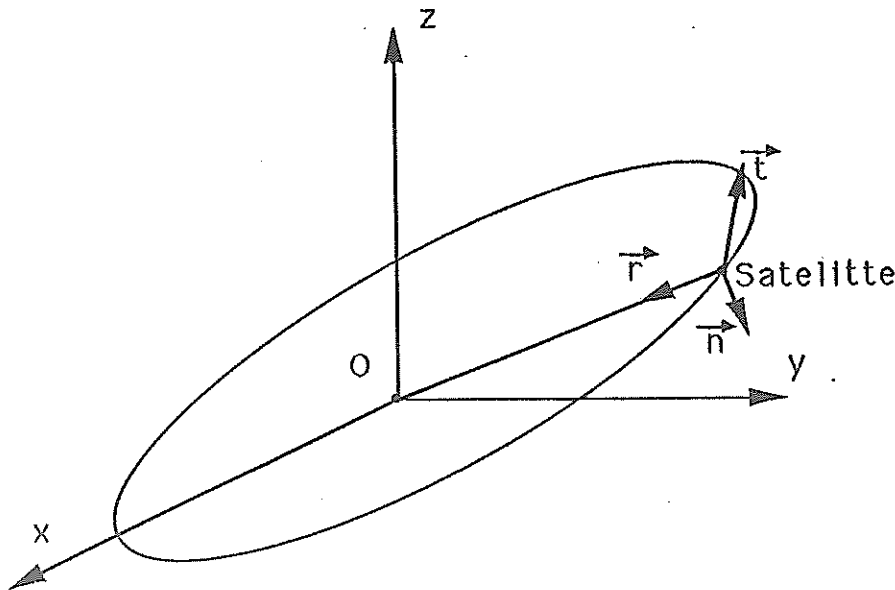
$$\frac{de_y}{dt} = \frac{2.T}{V} \cdot \sin \alpha - \frac{R}{V} \cdot \cos \alpha$$

$$\frac{di}{dt} = -\cos \alpha \cdot \frac{N}{V}$$

$$\frac{d\Omega}{dt} = -\frac{\sin \alpha}{\sin i} \cdot \frac{N}{V}$$

$$\frac{d\alpha}{dt} = n - \frac{R}{V} + \frac{\sin \alpha}{\tan i} \cdot \frac{N}{V}$$

where T, N, R are the components of the perturbation forces in the satellite coordinate system (see the following figure).



The coordinate system (t, n, r) is defined by :

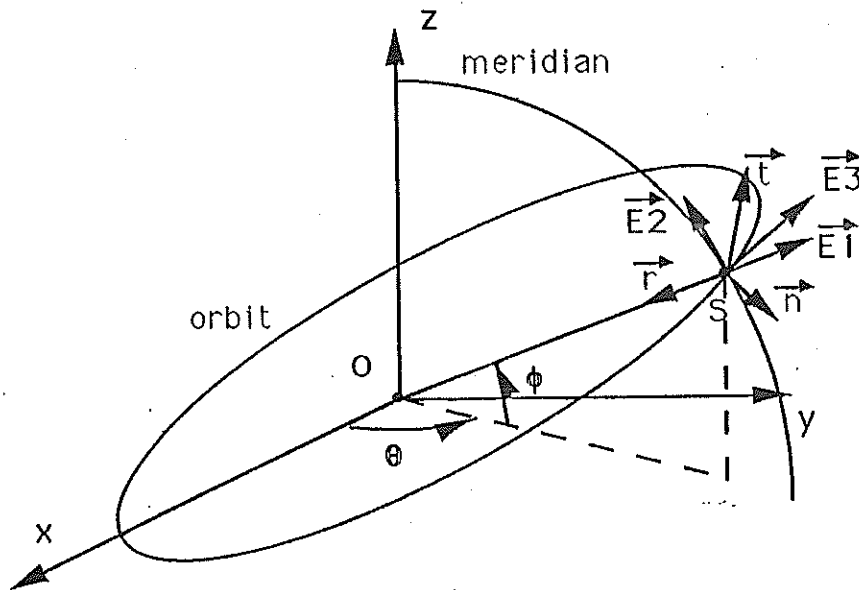
- # origin : S located at the mass center of the satellite
- # r axis : from S to the Earth mass center.
- # n axis : opposite of the angular momentum vector of the osculating orbit
- # t axis : orthonormal to r and n
(in the direction of the velocity vector, when $e=0$)

Therefore, we are going to apply this formula in the case of the J2 perturbation. The first computation is to project in the (t, n, r) coordinate system the acceleration due to the J2 term.

Computation of the acceleration due to J2 term

Acceleration in a spherical coordinate system :

We consider the following spherical system $(E1, E2, E3)$:



The coordinate system $(E1, E2, E3)$ is defined by :

- # origin : S located at the mass center of the satellite
- # E1 axis : from the Earth mass center to S.
- # E2 axis : along the meridian.
- # E3 axis : orthonormal to E1 and E2.

Then, the perturbing potential of J2 is :

$$U_{\text{per}} = -\mu \cdot \frac{a_e^2}{r^3} \cdot \frac{J_2}{2} \cdot (3 \cdot \sin^2 \phi - 1)$$

and the corresponding acceleration is :

$$\gamma = \text{grad } U_{\text{per}}$$

hence :

$$\begin{aligned} \gamma &= \frac{\partial U_{\text{per}}}{\partial r} \cdot \mathbf{E1} + \frac{1}{r} \cdot \frac{\partial U_{\text{per}}}{\partial \phi} \cdot \mathbf{E2} \\ &= 3 \cdot \mu \cdot \frac{a_e^2}{r^4} \cdot \frac{J_2}{2} \cdot [(3 \cdot \sin^2 \phi - 1) \cdot \mathbf{E1} - 2 \cdot \sin \phi \cdot \cos \phi \cdot \mathbf{E2}] \end{aligned}$$

Acceleration components T, N, R :

Now we have to change from the spherical system to the satellite system, knowing that :

$$\mathbf{E1} = r$$

$$\mathbf{E2} = t \cdot \cos J - n \cdot \sin J$$

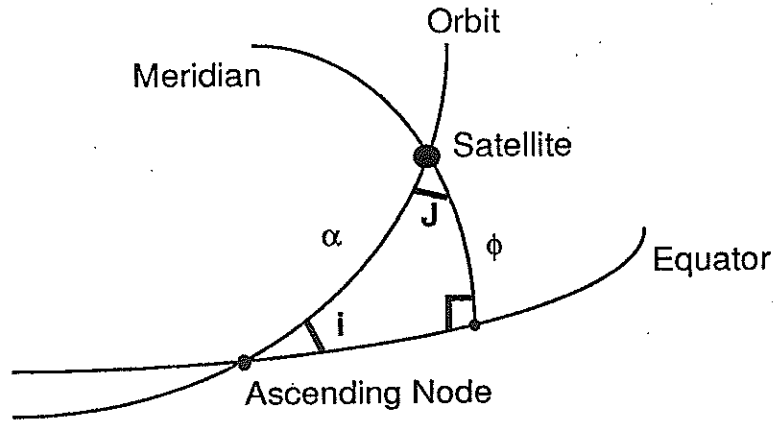
where J is the complementary angle between the normal of the orbit plan and the North.

We consider the spherical triangle below, and we apply the formula of spherical trigonometry :

$$\sin \phi = \sin \alpha \cdot \sin i$$

$$\sin J = \frac{\cos i}{\cos \phi}$$

$$\cos J = \frac{\sin \phi}{\cos \phi} \cdot \frac{\cos \alpha}{\sin \alpha}$$



Then, we can eliminate J in the acceleration, and we have :

$$\gamma = -3\mu \cdot \frac{a_e^2}{r^4} \cdot \frac{J_2}{2} \cdot \left[(1 - 3\sin^2 \phi) \cdot r + 2\sin^2 \phi \cdot \frac{\cos \alpha}{\sin \alpha} \cdot t - 2\sin \phi \cdot \cos i \cdot n \right]$$

and knowing that :

$$\sin^2 \phi = \sin^2 i \cdot \frac{(1 - \cos 2\alpha)}{2}$$

we get the perturbation accelerations :

$$N = \frac{3}{4} \cdot \mu \cdot \frac{a_e^2}{a^4} \cdot J_2 \cdot (3\sin^2 i - 2 - 3\sin^2 i \cdot \cos 2\alpha)$$

$$T = -\frac{3}{2} \cdot \mu \cdot \frac{a_e^2}{a^4} \cdot J_2 \cdot \sin^2 i \cdot \sin 2\alpha$$

$$N = \frac{3}{2} \cdot \mu \cdot \frac{a_e^2}{a^4} \cdot J_2 \cdot \sin 2i \cdot \sin \alpha$$

Gauss Equations

Now we can write the Gauss system for the J2 perturbation

$$\frac{da}{dt} = -3.n.\frac{a_e^2}{a} . J_2 . \sin^2 i . \sin 2\alpha$$

$$\frac{de_x}{dt} = -\frac{3}{2} . n . \left(\frac{a_e}{a}\right)^2 . J_2 . \left[\left(1 - \frac{5}{4} . \sin^2 i\right) . \sin \alpha + \frac{7}{4} . \sin^2 i . \sin 3\alpha \right]$$

$$\frac{de_y}{dt} = -\frac{3}{2} . n . \left(\frac{a_e}{a}\right)^2 . J_2 . \left[\left(\frac{7}{4} . \sin^2 i - 1\right) . \cos \alpha - \frac{7}{4} . \sin^2 i . \cos 3\alpha \right]$$

$$\frac{di}{dt} = -\frac{3}{4} . n . \left(\frac{a_e}{a}\right)^2 . J_2 . \sin 2i . \sin 2\alpha$$

$$\frac{d\Omega}{dt} = -\frac{3}{2} . n . \left(\frac{a_e}{a}\right)^2 . J_2 . \cos i . (1 - \cos 2\alpha)$$

$$\frac{d\alpha}{dt} = n . \left[1 - \frac{3}{2} . \left(\frac{a_e}{a}\right)^2 . J_2 . (4 . \sin^2 i - 3 - (1 + 2 . \sin^2 i) . \cos 2\alpha) \right]$$

with $\alpha = n.t$

Now, if we want to compute the variations along a certain period, we have to integrate these equations, which can be done easily :

$$a = a_0 + \frac{3}{2} . \frac{a_e^2}{a_0} . J_2 . \sin^2 i_0 . \cos 2\alpha$$

$$e_x = \frac{3}{2} . \left(\frac{a_e}{a_0}\right)^2 . J_2 . \left[\left(1 - \frac{5}{4} . \sin^2 i_0\right) . \cos \alpha + \frac{7}{12} . \sin^2 i_0 . \cos 3\alpha \right] + e_{x_0}$$

$$e_y = \frac{3}{2} . \left(\frac{a_e}{a_0}\right)^2 . J_2 . \left[\left(1 - \frac{7}{4} . \sin^2 i_0\right) . \sin \alpha + \frac{7}{12} . \sin^2 i_0 . \sin 3\alpha \right] + e_{y_0}$$

$$i = i_0 + \frac{3}{8} . \left(\frac{a_e}{a_0}\right)^2 . J_2 . \sin 2i_0 . \cos 2\alpha$$

$$\Omega = \Omega_0 - \frac{3}{2} \cdot \left(\frac{a_e}{a_0}\right)^2 \cdot J_2 \cdot \cos i_0 \cdot \left(\alpha - \frac{\sin 2\alpha}{2}\right)$$

Numerical application :

$$\begin{aligned} J_2 &= 1.082627 \times 10^{-3} \\ a_e &= 6378.137 \times 10^3 \text{ m} \\ a_0 &= 7205 \times 10^3 \text{ m} \end{aligned}$$

hence :

$$\begin{aligned} a &= 7205 + 9.17 \cos 2\alpha \\ e_x &= -3.27 \cdot 10^{-4} - 2.82 \cdot 10^{-4} \cos \alpha + 7.25 \cdot 10^{-4} \cos 3\alpha \\ e_y &= 1.47 \cdot 10^{-3} - 9.03 \cdot 10^{-4} \sin \alpha + 7.25 \cdot 10^{-4} \cos 3\alpha \\ i &= 98.7 - 5.45 \cdot 10^{-3} \cos 2\alpha \\ \Omega &= 249.7 + 1.10 \cdot 10^{-2} \alpha - 5.51 \cdot 10^{-3} \sin 2\alpha \end{aligned}$$

Conclusion

The consistency of the computed results with the theoretical results taking into account only the J2 perturbation is really good (all the magnitude are almost the same and only the initial values are slightly different). Therefore, we can conclude that the variations of the orbital parameters along one orbit are mainly due to the J2 perturbation.

Good

Influence of surface forces on the orbital parameters

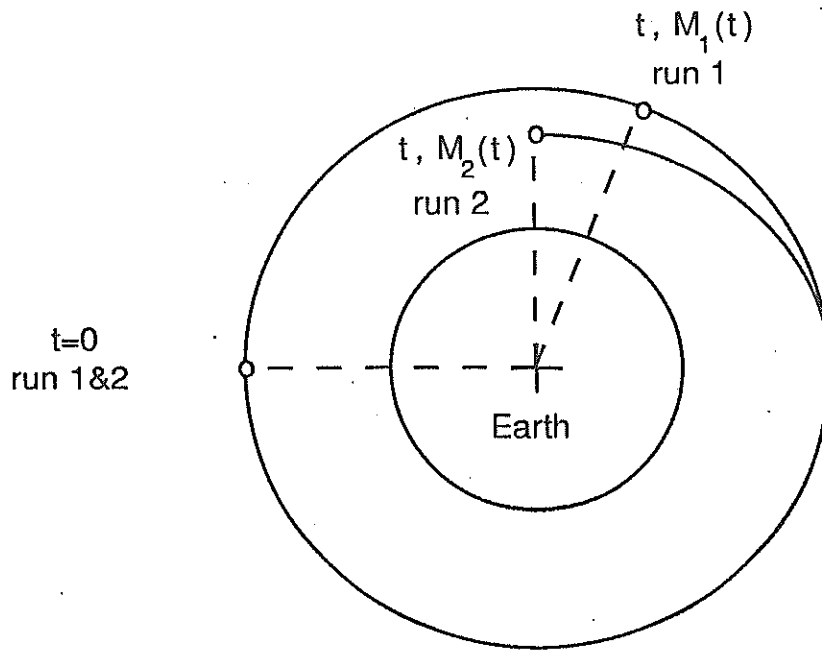
Method used

At first, we ran UTOPIA in two cases

- with all the perturbation forces (run 1)
- with all the perturbation forces except the atmospheric drag (run 2)

In each case, we had outputs giving the variations of the orbital parameters along one orbit. So, we substracted these two kinds of data in the following manner :

$$\begin{aligned} \Delta a(t) &= a_2(t) - a_1(t) \\ \Delta e(t) &= e_2(t) - e_1(t) \end{aligned}$$



Position of the satellites corresponding to run 1&2

But because of the different forces applied in the two cases, the evolution of the orbital parameters as a function of the time t is different in each case. This is especially true for the mean anomaly, respectively $M_1(t)$ and $M_2(t)$ for the run 1 and 2.

$$\Delta a(t) = a_2(M_2(t)) - a_1(M_1(t))$$

$$\Delta e(t) = e_2(M_2(t)) - e_1(M_1(t))$$

.....

As the orbital parameters vary along one orbit (and therefore as a function of the mean anomaly), the calculated differences $\Delta x(t)$ included not only the influence of the atmospheric drag but also the variation of the orbital parameters along the trajectory.

Therefore, in order to avoid this effect, the comparison has been done between a model without perturbation forces (only a central force in $1/r^2$) and a model taking into account only the atmospheric drag as perturbation force (and always the central force in $1/r^2$). In the first model without perturbation force, the orbital parameters are constant and therefore, there are no problem anymore.

Theoretical study

Introduction

A very simple and rapid method is developed to study the effect of the atmospheric drag perturbation on the low & circular orbit of a satellite. For that purpose we will use Gauss equations established for a circular orbit, and the results of the forces computation. That method explains the shape of the variation of the parameters.

In a second time we have used the orbit propagator Utopia to determine the influence of the atmospheric drag perturbation on the Kepler parameters. The atmospheric density has been determined by the semi-empirical density model Jacchia 1971, Jacchia 1977 and DTM. For each of them we have propagated the orbit for different case of solar activity and geomagnetic activity.

Method

We will use the same assumptions as the J2 study :

$$r \approx a$$

$$e \approx 0$$

$$V \approx a.n = \text{constant}$$

$$\alpha_v = \omega + v \approx \alpha = \omega + M$$

Which gives the Gauss system, for a circular orbit :

$$\frac{da}{dt} = 2.a.\frac{T}{V}$$

$$\frac{de_x}{dt} = \frac{2.T}{V} \cdot \cos \alpha + \frac{R}{V} \cdot \sin \alpha$$

$$\frac{de_y}{dt} = \frac{2.T}{V} \cdot \sin \alpha - \frac{R}{V} \cdot \cos \alpha$$

$$\frac{di}{dt} = -\cos \alpha \cdot \frac{N}{V}$$

$$\frac{d\Omega}{dt} = -\frac{\sin \alpha}{\sin i} \cdot \frac{N}{V}$$

$$\frac{d\alpha}{dt} = n - \frac{R}{V} + \frac{\sin \alpha}{\tan i} \cdot \frac{N}{V}$$

where T, N, R are the components of the perturbation forces in the satellite coordinate system.

As before, we are going to apply this formula in the case of the atmospheric drag perturbation. The first computation is to give a model in the (t, n, r) coordinate system of the acceleration due to the atmospheric drag.

Acceleration model of the atmospheric drag

Results of the Forces computation :

Drag accelerations are computed by the following formula in the coordinate system (t, n, r) :

$$T = \frac{1}{2} \rho \cdot \frac{V^2}{m} \cdot (S.C_d)$$

$$N = \frac{1}{2} \rho \cdot \frac{V^2}{m} \cdot (S.C_{ln})$$

$$R = \frac{1}{2} \rho \cdot \frac{V^2}{m} \cdot (S.C_{lr})$$

As one can see, the acceleration is dependent with drag coefficients S.C_i and the atmospheric density ρ.

The atmospheric density can be modelized along one orbit by :

$$\rho = \rho_0 + \rho_1 \sin (\alpha + \phi_0)$$

(the equi-atmospheric density surface have an ovoïd shape that we can called solar bubble)

and the drag coefficients by (see appendix R) :

$$S.C_d = S_b.C_{db} + S_a.C_{da} | \cos (\alpha + \phi_1) |$$

$$S.C_{ln} = S_a.C_{lna} \cos (\alpha + \phi_2)$$

$$S.C_{lr} = S_a.C_{lra} \cos (2\alpha + \phi_3)$$

where :

ρ_0 is the mean atmospheric density along one orbit

ρ_1 is the amplitude of the variation of the density

S_b is the surface of the box of the satellite

C_{db} is the drag coefficient for the box

S_a is the surface of the solar array

C_{da} , C_{lna} , C_{lra} are the drag and lift coefficients of the solar array

Influence on the semi-major axis and the eccentricity :

In a first approximation one can neglect the Normal and Radial accelerations because :

$$|S.C_{ln}| \ll |S.C_d| \quad \text{and} \quad |S.C_{lr}| \ll |S.C_d|$$

hence : $N \approx 0$ and $R \approx 0$.

Therefore the influence of atmospheric drag is mainly on the semi-major-axis and the eccentricity, and the Gauss equations are :

$$\frac{da}{dt} = 2a \frac{T}{V} = - \frac{a.V}{m} (\rho_0 + \rho_1 \sin (\alpha + \phi_0)) . (S_b.C_{db} + S_a.C_{da} | \cos (\alpha + \phi_1) |)$$

$$\frac{de_x}{dt} \approx 2 \cdot \frac{T}{V} \cos \alpha$$

$$\frac{de_x}{dt} \approx - \frac{V}{m} \cos \alpha (\rho_0 + \rho_1 \sin (\alpha + \phi_0)) . (S_b.C_{db} + S_a.C_{da} | \cos (\alpha + \phi_1) |)$$

$$\frac{de_y}{dt} \approx 2 \cdot \frac{T}{V} \sin \alpha$$

$$\frac{de_y}{dt} \approx -\frac{V}{m} \sin \alpha (\rho_0 + \rho_1 \sin(\alpha + \phi_0)) \cdot (S_b \cdot C_{db} + S_a \cdot C_{da} |\cos(\alpha + \phi_1)|)$$

that we can integrate it easily, keeping only the secular effect and one gets the secular decrease of the semi-major axis :

$$\Delta a_{sec} = \frac{a \cdot V}{m} \cdot \rho_0 \cdot (S_b \cdot C_{db} + \frac{2}{\pi} S_a \cdot C_{da}) \Delta t$$

and the decrease of the eccentricity :

$$(\Delta e_x)_{sec} = \frac{\rho_1 \cdot V}{2m} (S_b \cdot C_{db} + \frac{2}{\pi} S_a \cdot C_{da}) \sin \phi_0 \cdot \Delta t$$

$$(\Delta e_y)_{sec} = \frac{\rho_1 \cdot V}{2m} (S_b \cdot C_{db} + \frac{2}{\pi} S_a \cdot C_{da}) \cos \phi_0 \cdot \Delta t$$

$$(\Delta e)_{sec} = \frac{\rho_1 \cdot V}{2m} (S_b \cdot C_{db} + \frac{2}{\pi} S_a \cdot C_{da}) \Delta t$$

So we can notice that the secular change of the semi-major axis is proportional to the mean-density ρ_0 and to the drag coefficients C_{da} and C_{db} .

Similarly the secular change of the eccentricity is proportional to the amplitude variation of the density ρ_1 and to the drag coefficients C_{da} and C_{db} .

For one day, the decrease of the semi-major axis is about a few meters and the eccentricity a few millionths.

influence on the inclination and the argument ascending node :

A same development can be made to study the variation of the inclination and the argument of the ascending node considering the Normal perturbation.

Computation [Ref 6.7,8]

The largest unknown parameter associated with the calculation of the atmospheric drag on a satellite is in the estimate of the atmospheric density. In order to compute the atmospheric density in our orbit propagation, we have used the semi-empirical density models : Jacchia 1971 (J71), Jacchia 1977 (J77) and DTM. But nevertheless, we have to keep in mind

that any of these state-of arts models can be in error anywhere from 10% during a period when the solar activity is minimal to 200% when the solar activity is maximal. Thus, the accuracy of the predicted positions are limited by the accuracy of the chosen atmospheric model and by the accuracy of the factors used to compute the density in these models. Eight primary factors are known to have effects on the density of the atmosphere, these can be classified as follows :

1. Altitude
2. Variation with the solar activity
3. Variation with the geomagnetic activity
4. The diurnal variation
5. Semi-annual variation
6. Seasonal-latitude variations of the lower thermosphere
7. Seasonal-latitude variations of helium
8. Rapid density fluctuations probably connected with gravity waves

All of these variations except the last type are subject to some amount of regularity and are interpolated in the atmospheric models.

We had focused our study on the comparison of the three models for different cases of solar and geomagnetic activities :

The *solar activity* has an important influence which produces disturbances in the upper atmosphere. The primary sources of these disturbances are solar flares and solar plasma events. The correlation of these phenomena is associated with the 27-day solar rotation period and the 11-year solar cycle. The 10.7 cm solar flux index, *sof*, and the 10.7cm smooth solar flux (over a 164-day period) are used as indexes of solar extreme ultraviolet radiation which heats up the Earth's atmosphere in the atmospheric density models.

The *geomagnetic activity* and interaction with the charged particles in the upper atmosphere cause a density wave propagation from high to low magnetic latitudes. The effect of the geomagnetic activity is most pronounced near the geomagnetic pole and decreases when approaching to the Equator. The planetary geomagnetic index, *Kp*, is used as a measure of the geomagnetic activity.

In the appendix V, we have drawn the variation of the solar flux, sof, the smooth solar flux, ssof, and the geomagnetic index, Kp, from January 1976 to September 1989 to have a global view of the changes, and from January 1989 to September 1989 to have a more precise idea of the local changes.

Results

First we studied the influence of atmospheric drag using the three different density model J71, J77 and DTM, with the geomagnetic index and solar flux indexes corresponding to June 23, 1989. For that day we had :

$$\text{ssof} = 206.0 , \text{sof} = 233.7 , \text{Kp} = 1.3$$

which correspond to a high solar activity and a low geomagnetic activity. The results on a 24 hour length of time with the atmospheric model DTM are given in appendix W. For each case, we have listed the input deck for the orbit propagator Utopia, plotted the temperature, the atmospheric density, the drag and lift coefficients, the atmospheric drag accelerations along one orbit and finally the influences on the Kepler parameters.

The results we obtained support perfectly the theory we have previously developed. The most important influence of the atmospheric drag is the decrease of the semi-major axis, and to a lesser degree the decrease in the eccentricity. The following table we have put some results of the orbit-propagator Utopia, using different values of smooth solar flux, ssof, the solar flux, sof, and the geomagnetic index Kp. We can notice that the change can be of the order of 30 depending on the value of the 3 parameters (for a similar altitude) (see appendix X).

Then for each density model we looked at the decrease, Δa , of the semi-major axis for a one-day orbit of SPOT.

The density was computed for different cases of geomagnetic index and solar flux : for low geomagnetic activity, Kp was taken as equal to 1.0, for medium activity to 4.0, and for magnetic storms to 7.0 ; for solar flux indexes we have considered 3 cases for the smooth solar flux, ssof : 100.0, 150.0, and 200.0, and 3 different cases for the solar flux, sof :
 $\text{sof} = \text{ssof} - 50.0$, $\text{sof} = \text{ssof}$, and $\text{sof} = \text{ssof} + 50.0$

The results are listed in appendix Y.

Influence of the thermal forces on the orbital parameters

Method used

As exposed previously in III.1, the influence study has been done without all the others perturbation forces.

Results

The calculations have been done for a 24 hour time length. The influence of the thermal forces on a , e , i , ω , Ω , $\omega+f$ and M are presented in appendix Z. The main effects are :

- a decrease of the eccentricity ($-0.75 \cdot 10^{-6}$ / 24 hours)
- an increase of the inclination ($+0.010$ arcsec/ 24 hours)
- an increase of the ω ($+85$ arcsec/ 24 hours)
- a decrease of Ω ($-6.0 \cdot 10^{-3}$ arcsec/ 24 hours)
- a decrease of the mean anomaly (-0.024 deg/ 24 hours)

Theoretical calculations

Some of these results can be found easily using the Gauss equations for a circular orbit.

Semi-major axis

$$\frac{da}{dt} = 2a \frac{T}{V} \quad \text{hence} \quad a(t) = a_0 + \int_0^t \frac{2a}{V} T(t) dt$$

$$a(t) \approx a_0 + \frac{2a_0}{V} \int_0^t T(t) dt$$

But $\int_0^\tau T(t) dt \approx 0$ with τ being the period of the satellite

Therefore, there is no noticeable secular effect on the semi-major axis and just a variation which is almost sinusoidal.

Inclination

$$\frac{di}{dt} = -\frac{N}{V} \cos \alpha \quad \text{hence} \quad i(t) = i_0 - \frac{1}{V} \int_0^t N(t) \cos(nt + \omega_0) dt$$

$$\begin{aligned} \int_0^\tau N(t) \cos(nt + \omega_0) dt &\approx 2.0 \cdot 10^{-8} \int_0^{240} \frac{\cos(\theta + \omega_0)}{n} d\theta \\ &\approx \frac{2.0 \cdot 10^{-8}}{n} \int_{\omega_0}^{240 + \omega_0} \cos \theta d\theta \end{aligned}$$

This approximation has been done because the acceleration $N(t)$ is almost constant for an orbit angle between 0° to 240° ($\approx 20 \text{ nms}^{-2}$) and then almost equal to zero.

Numerical application :

$$V = 7440 \text{ ms}^{-1}$$

$$\omega_0 = 102^\circ$$

$$n = 2\pi/\tau$$

$$\tau = 6086 \text{ s}$$

$$i(t) \approx 98.7 + 3.25 \cdot 10^{-11} t \quad (i \text{ in deg, } t \text{ in sec})$$

This formula takes only into account the secular effect on the inclination. Therefore, there is a secular variation of the inclination of the order of $0.010 \text{ arcsec}/24 \text{ hours}$ and a slight sinusoidal variation.

Results

(secular variations)

Atmospheric drag Thermal forces

$\Delta a/24h$	-3.75 m	-
$\Delta e/24h$	$-0.075 \cdot 10^{-6}$	$-0.75 \cdot 10^{-6}$
$\Delta i/24h$	$-9.0 \cdot 10^{-4}$ arcsec	$+1.0 \cdot 10^{-2}$ arcsec
$\Delta \omega/24h$	-13.0 arcsec	+85 arcsec
$\Delta \Omega/24h$	$-1.8 \cdot 10^{-3}$ arcsec	$-6.0 \cdot 10^{-3}$ arcsec
$\Delta M/24h$	+ 0.0056 deg	-0.024 deg

Conclusion

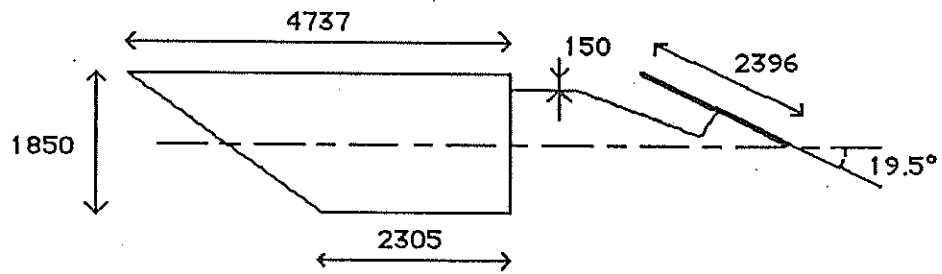
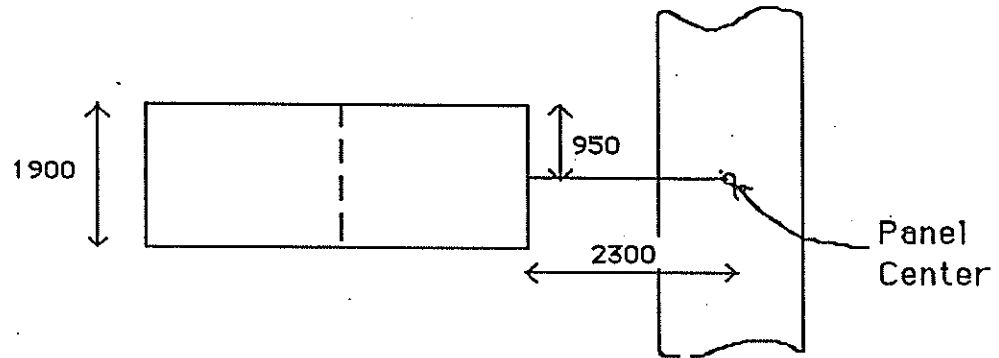
Our project at CCAR has been a great opportunity for us to implement our knowledge on a real project with the guidance of experienced researchers. Moreover, the excellent work atmosphere discovered in the research center makes us eager to enter the industrial world and cope with new challenges.

References

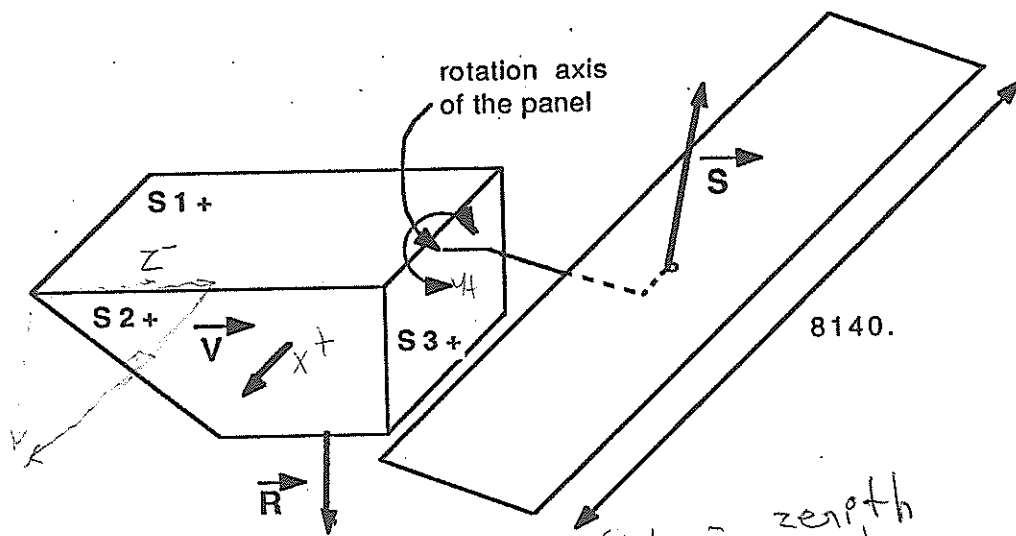
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Appendices

2D - Trapezoidal model of SPOT



3D - TRAPEZOIDAL MODEL OF SPOT

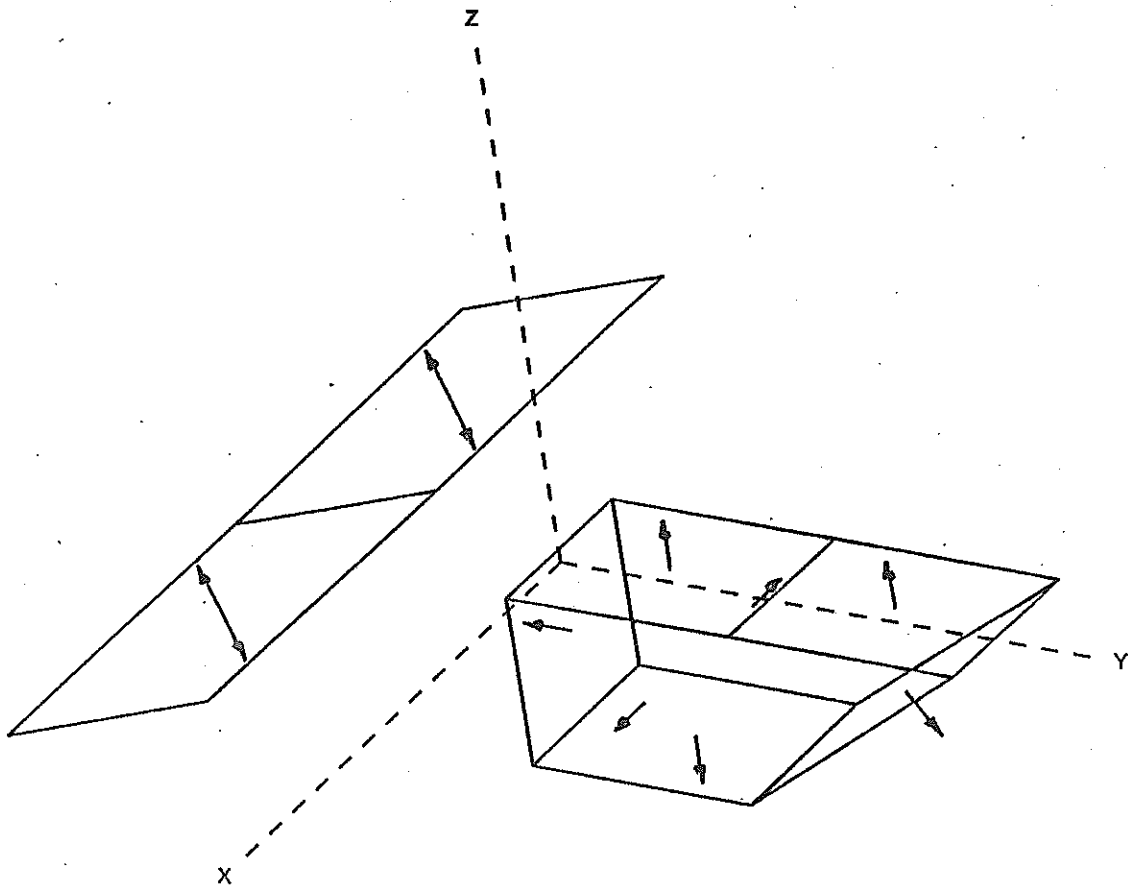


S_{3+} = flight direction
 S_{2-} = rear of flight direction

S_{1+} = zenith
 S_{1-} = nadir
 S_{2+} = normal to velocity vector
 S

\vec{V} : points the direction of the velocity
 \vec{R} : points the direction of Earth's center
 \vec{S} : points the direction of the sun

$$\begin{aligned}
 S_{1-} &= +Z \\
 S_{1+} &= -Z \\
 S_{2+} &= \vec{x} \\
 S_{2-} &= +X \\
 S_{3+} &= -y \\
 S_{3-} &= ay + bz
 \end{aligned}$$



VIEW = 3-D
SCALE = 0.1727
NV = 1

I.R. optical properties

Face	epsilon	tau	KS	KD
S1+	0.65	0.91	0.32	0.03
S1-	0.76	0.88	0.21	0.03
S2+	0.76	0.88	0.21	0.03
S2-	0.76	0.88	0.21	0.03
S3+	0.76	0.87	0.21	0.03
S3-	0.75	0.90	0.22	0.03
SP+	0.85	0.50	0.08	0.07
SP-	0.82	0.58	0.10	0.08

epsilon

absorbed component

KS = (1-epsilon) tau

specularly reflected component

KD = (1-epsilon) (1-tau)

diffusively reflected component

Visible optical properties

Face	alpha	tau	KS	KD
S1+	0.40	0.91	0.55	0.05
S1-	0.40	0.88	0.53	0.07
S2+	0.39	0.88	0.54	0.07
S2-	0.39	0.88	0.54	0.07
S3+	0.36	0.87	0.56	0.08
S3-	0.45	0.90	0.50	0.05
SP+	0.69	0.50	0.16	0.16
SP-	0.51	0.58	0.28	0.21

alpha

absorbed component

KS = (1-alpha) tau

specularly reflected component

KD = (1-alpha) (1-tau)

diffusively reflected component

Albedo and Emissivity of Earth can be represented as a spherical harmonic expansion

- Knocke's 2 degree zonal harmonic model:
 - Seasonally Varying
 - No longitudinal dependency

$$a = a_0 + a_1 P_1(\sin\phi) + a_2 P_2(\sin\phi)$$

$$\varepsilon = \varepsilon_0 + \varepsilon_1 P_1(\sin\phi) + \varepsilon_2 P_2(\sin\phi)$$

where

$P_1(\sin\phi)$, $P_2(\sin\phi)$ = Legendre polynomials

$$a_1 = c_0 + c_1 \cos(\omega(\text{JD}-t_0)) + c_2 \sin(\omega(\text{JD}-t_0)),$$

$$\varepsilon_1 = k_0 + k_1 \cos(\omega(\text{JD}-t_0)) + k_2 \sin(\omega(\text{JD}-t_0)),$$

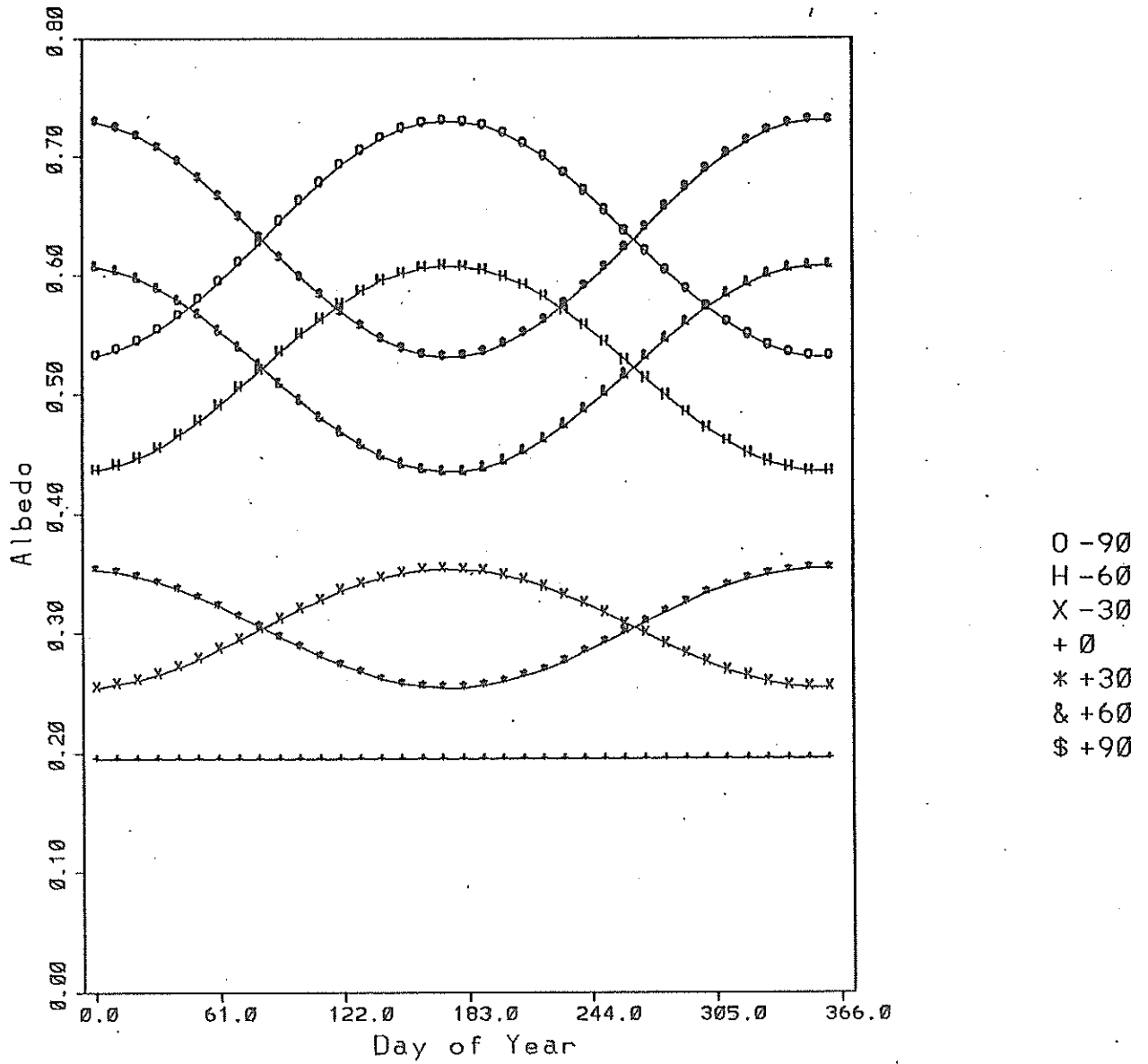
t_0 = the epoch of periodic terms,

ω = the orbital period of the earth,

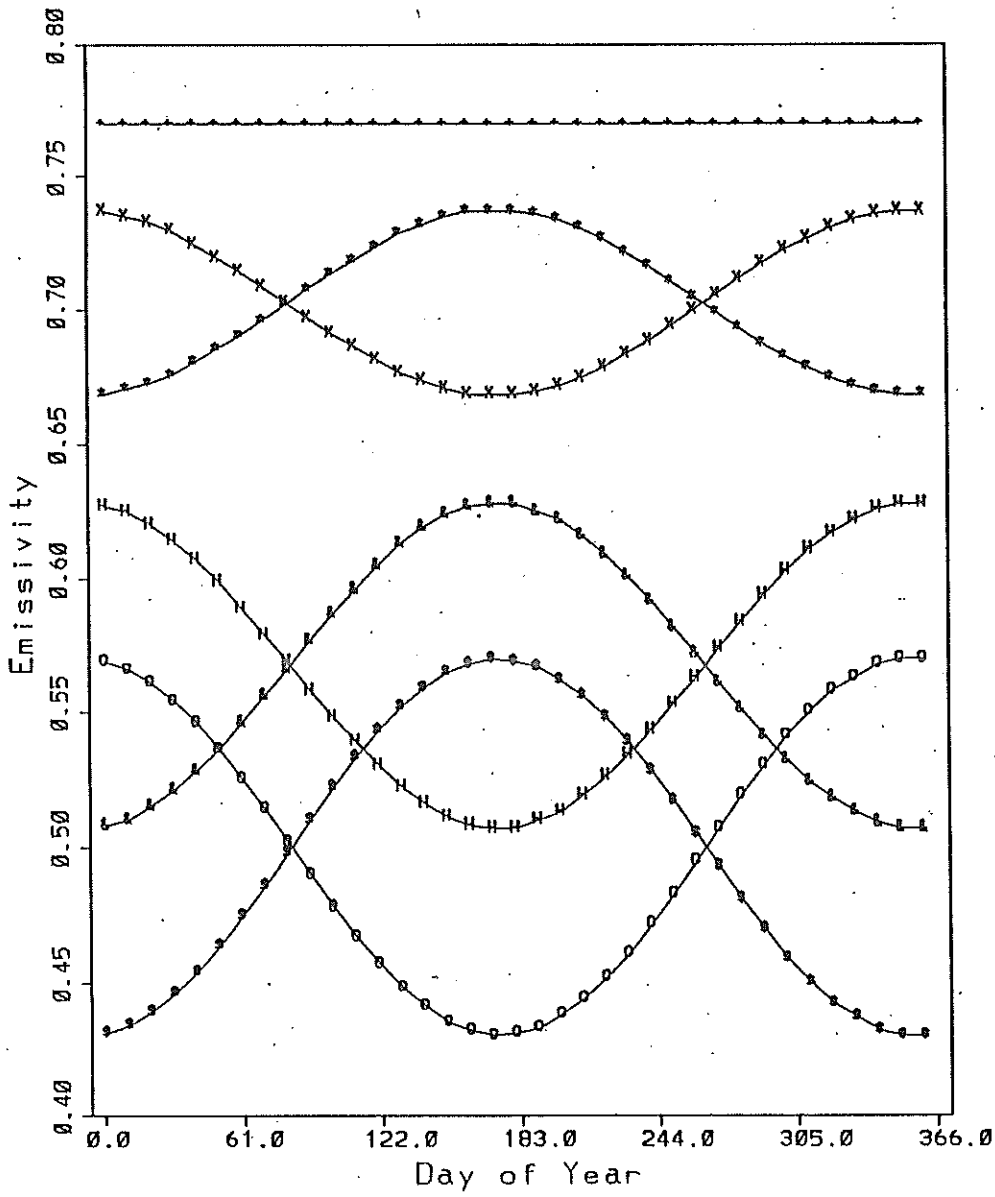
$a_0, c_0, \varepsilon_0, k_0, c_1, k_1, a_2, c_2, \varepsilon_2, k_2$ = constants

- Bess's 12 X 12 model from Nimbus-6 and Nimbus-7 Satellites

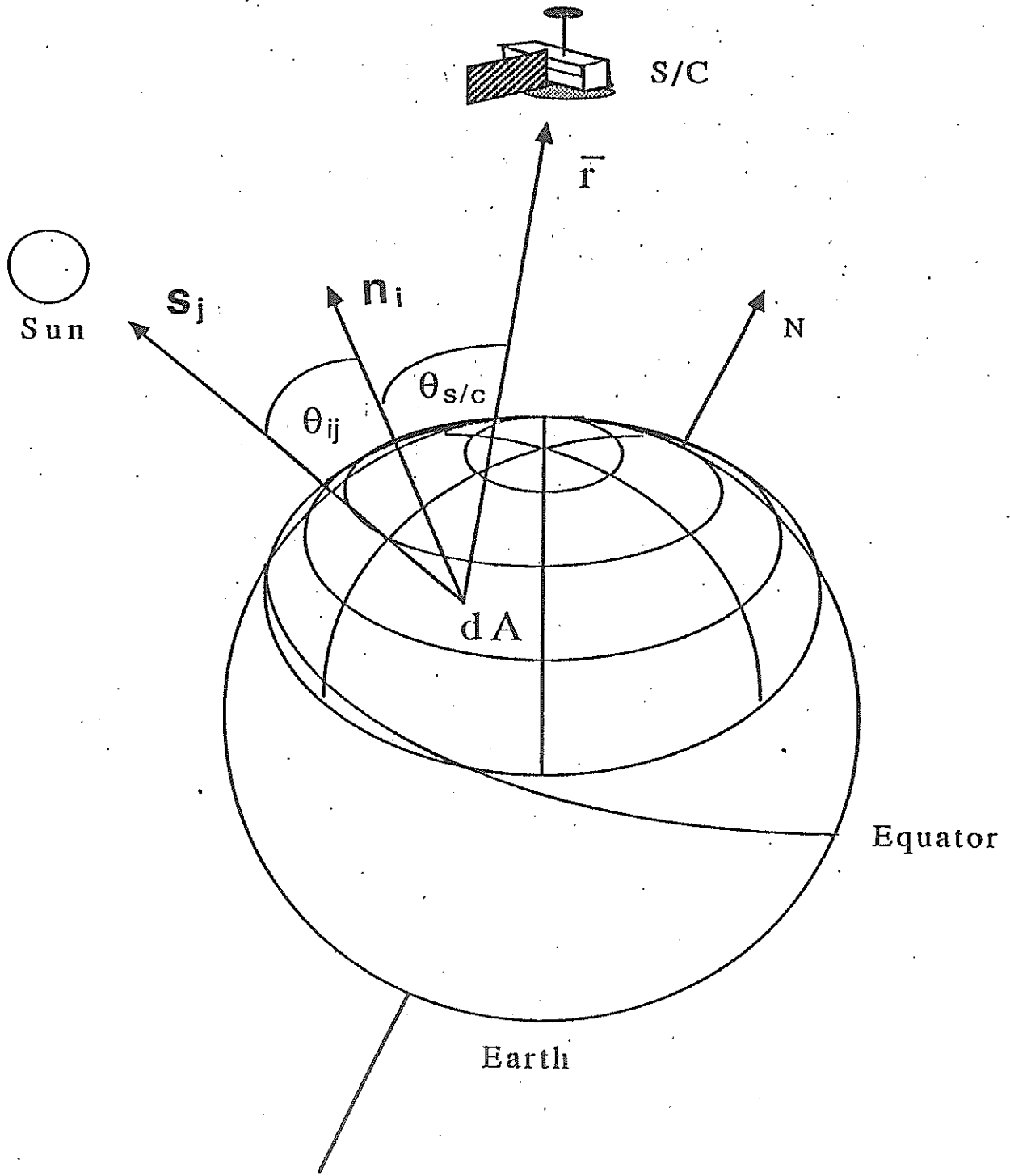
Albedo Vs. Day of Year



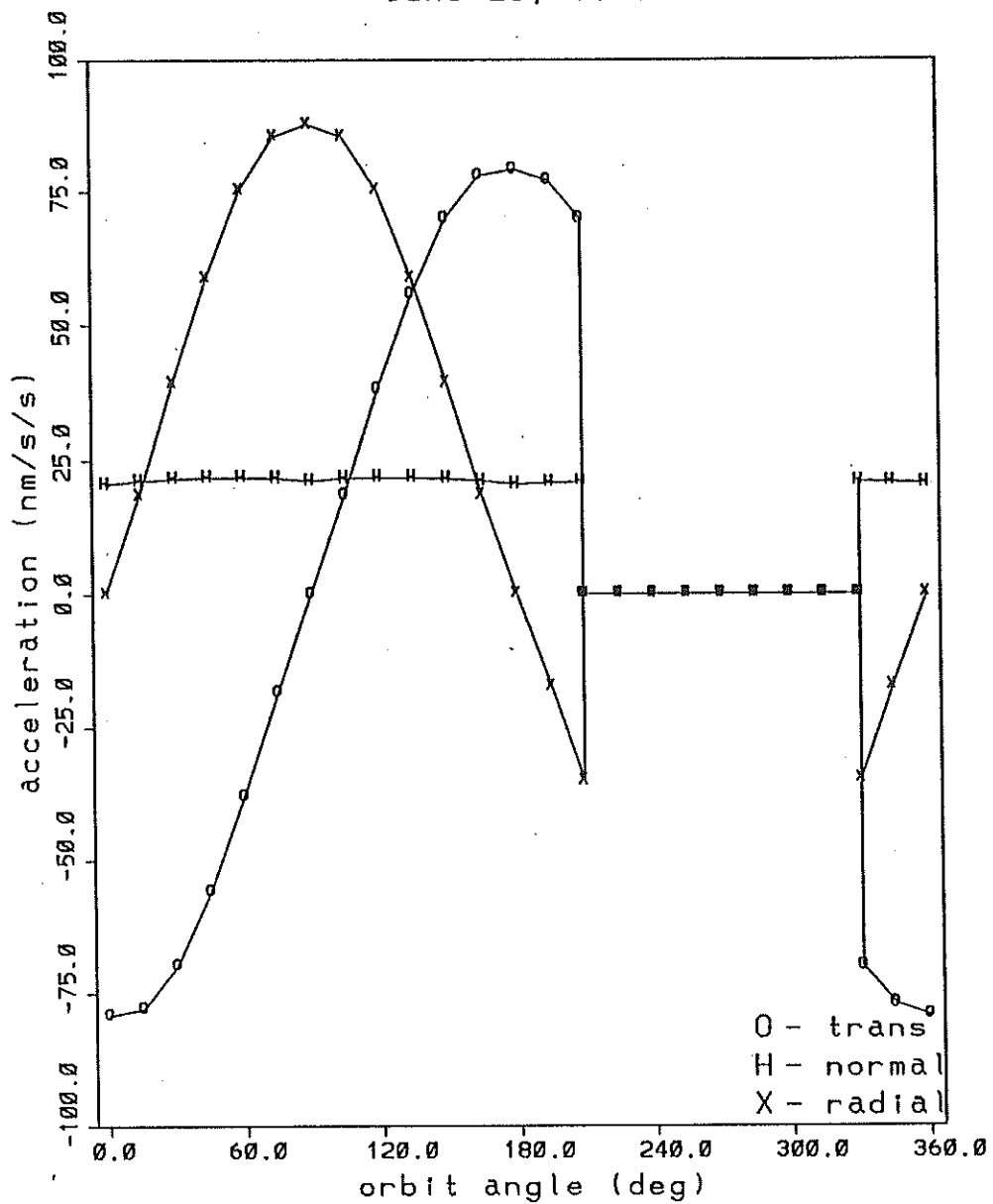
Emissivity Vs. Day of Year



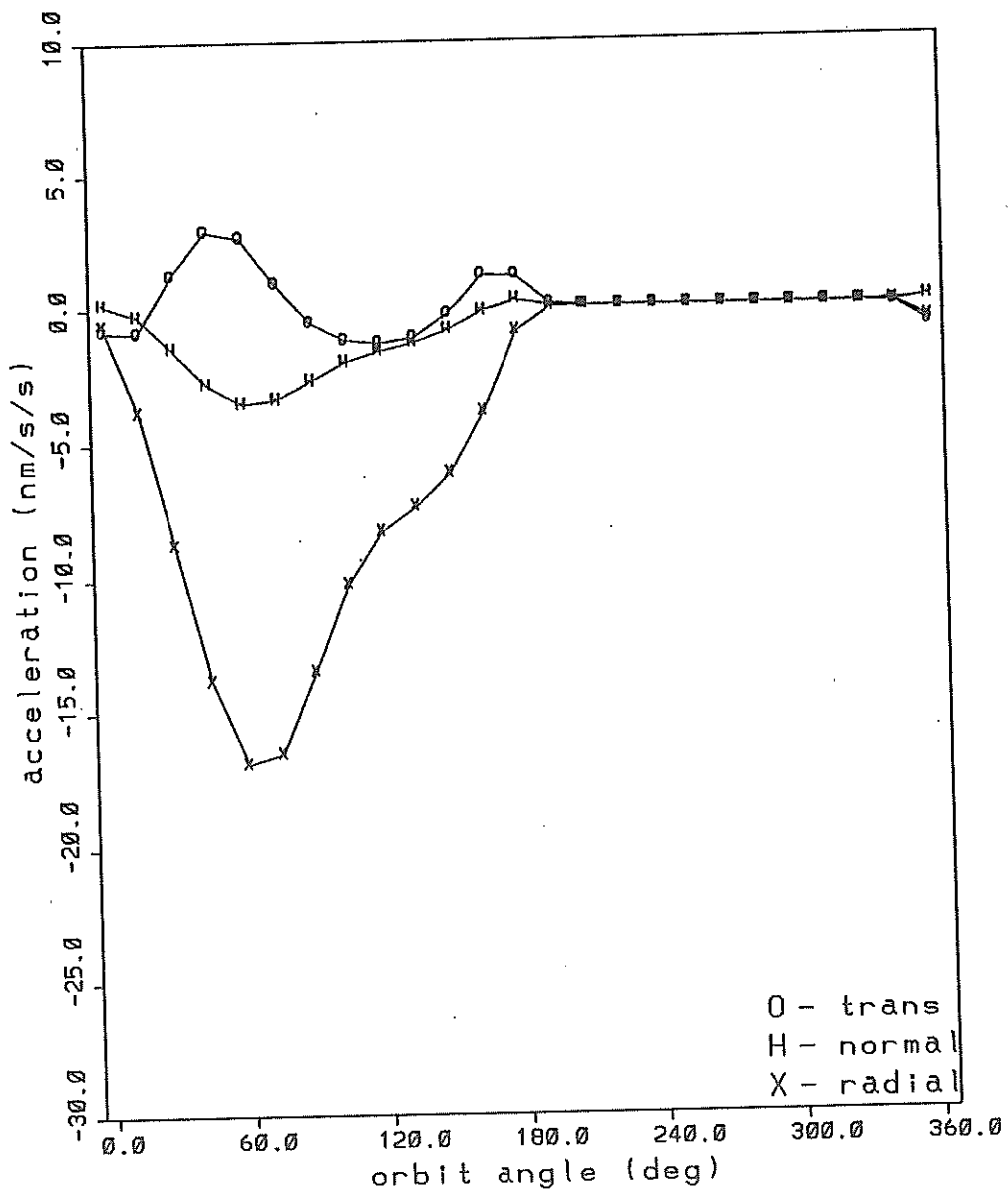
Earth Elemental Areas



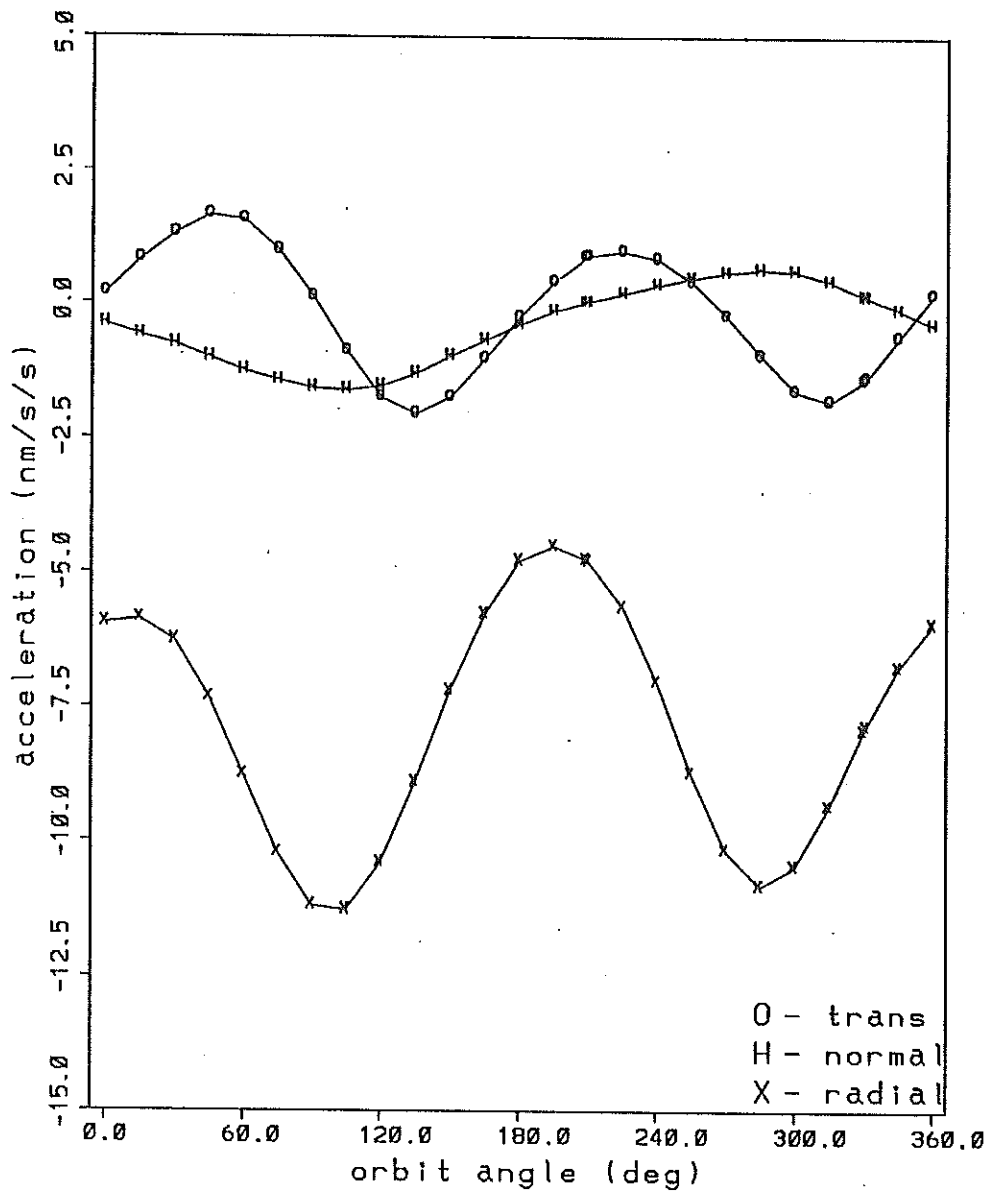
solar forces on spot
June 23, 1989



Albedo forces on spot
June 23, 1989



Plan. forces on spot
June 23, 1989



PROGRAM SATIR

F.1

This program calculates the accelerations due to the infrared
emission of the satellite

REAL H,W,CIGMA,C,SIG,SHADIN,SHADOUT,SUNANG
INTEGER I,J,K,L,M
DIMENSION A(12),E(12),P(12,3),F(12,35),T(12,35),ACC(35,3)
DIMENSION ANC(35)
OPEN(1,STATUS='OLD',FORM='FORMATTED',FILE='satir35.tnr')
OPEN(2,STATUS='OLD',FORM='FORMATTED',FILE='satir29.tnr')

*** Definition of the area of each surface ***

A(1)=0.514
A(2)=7.179
A(3)=1.308
A(4)=0.797
A(5)=3.583
A(6)=1.316
A(7)=5.194
A(8)=1.339
A(9)=5.171
A(10)=1.050
A(11)=2.465
A(12)=5.806

*** Definition of the emissivity of each surface ***

E(1)=0.15
E(2)=0.45
E(3)=0.23
E(4)=0.15
E(5)=0.45
E(6)=0.15
E(7)=0.45
E(8)=0.15
E(9)=0.45
E(10)=0.15
E(11)=0.45
E(12)=0.45

*** Definition of the normal of each surface ***

DO 10 I=1,12
DO 20 K=1,3
P(I,K)=0.

CONTINUE

CONTINUE

P(1,3)=1.
P(2,3)=1.
P(3,3)=1.
P(4,3)=-1.
P(5,3)=-1.
P(6,1)=-1.
P(7,1)=-1.
P(8,1)=1.
P(9,1)=1.
P(10,2)=-1.
P(11,2)=-1.
P(12,2)=0.60543
P(12,3)=-0.79590

*** Check module of the normals of each surface ***

```

C
C DO 25 NC=1,12
C WRITE(*,*) 'Surface',NC
C DO 27 ND=1,3
C WRITE(*,*) 'P(',NC,',',',',ND,')=',P(NC,ND)
C27 CONTINUE
C25 CONTINUE
C
C *** Definition of constants ***
C
C PI: obvious, no....
C PI=3.14159
C C: Speed of light (m/s)
C C=3.00E+8
C SIG: Stephan-Boltzmann constant (W/m**2/K**4)
C SIG=5.67E-8
C W: weight of the satellite (kg)
C W=1850.
C SUNANG: angle between the periapsis and the orbital noon (deg.)
C SUNANG=49.825
C CIGMA: angle between the sunrise and the orbital noon (deg.)
C CIGMA=90.0
C SHADIN: angle of entry in the shade (deg.)
C SHADIN=169.00+90.-SUNANG
C SHADOU: angle of exit of the shade (deg.)
C SHADOUT=290.88+90.-SUNANG
C
C *** Definition of orbit point for calculations ***
C
C ANC(26)=SHADIN-0.1
C ANC(27)=SHADIN+0.1
C ANC(28)=SHADOUT-0.1
C ANC(29)=SHADOUT+0.1
C ANC(30)=CIGMA-0.1
C ANC(31)=CIGMA+0.1
C ANC(32)=CIGMA+89.9
C ANC(33)=CIGMA+90.1
C ANC(34)=CIGMA+269.9
C ANC(35)=CIGMA+270.1-360.
C
C *** Definition of the temperature of each surface ***
C
C DO 30 I=1,35
C IF (I.LT.25.5) THEN
C H=(I-1)*15.
C ANC(I)=H
C ELSE
C H=ANC(I)
C ENDIF
C WRITE(*,*) ANC(I)
C T(1,I)=305.+15*COS((H-CIGMA)*PI/180.)
C T(4,I)=282.5-7.5*COS((H-CIGMA)*PI/180.)
C T(6,I)=270.-20*SIN((H-CIGMA)*PI/180.)
C T(8,I)=270.+20*SIN((H-CIGMA)*PI/180.)
C T(5,I)=273.
C T(12,I)=188.
C IF ((H.GT.(CIGMA+90)).AND.(H.LT.(CIGMA+270))) THEN
C T(2,I)=188.
C T(3,I)=188.
C ELSE
C T(2,I)=338.
C T(3,I)=338.
C ENDIF
C IF ((H.GT.CIGMA).AND.(H.LT.SHADOUT)) THEN
C T(7,I)=188.
C ELSE

```



```

      T(7,I)=338.
    ENDIF
    IF ((H.GT.CIGMA).AND.(H.LT.SHADIN)) THEN
      T(9,I)=338.
    ELSE
      T(9,I)=188.
    ENDIF
    IF ((H.GT.SHADIN).AND.(H.LT.SHADOUT)) THEN
      T(10,I)=188.
      T(11,I)=188.
    ELSE
      T(10,I)=275.
      T(11,I)=338.
    ENDIF
30  CONTINUE
C
C  *** Check module of the temperatures ***
C
C  DO 75 NA=1,12
C    WRITE(*,*) 'surface',NA
C    DO 77 NB=1,35
C      WRITE(*,*) 'T(',ANC(NB),')=' ,T(NA,NB)
C77  CONTINUE
C75  CONTINUE
C
C  *** Calculation of the forces due to the emission of SPOT ***
C
C  DO 60 M=1,35
C    DO 70 J=1,12
C      F(J,M)=(-2*E(J)*A(J)*SIG*(T(J,M)**4))/(3*C)
70  CONTINUE
C    ACC(M,1)=0.
C    ACC(M,2)=0.
C    ACC(M,3)=0.
C    DO 80 L=1,12
C      DO 90 K=1,3
C        ACC(M,K)=ACC(M,K)+(F(L,M)*P(L,K)/W)
90  CONTINUE
80  CONTINUE
5  FORMAT(F10.1,3G17.7)
  WRITE(1,5) ANC(M),-ACC(M,1),ACC(M,2),-ACC(M,3)
  IF (M.LT.29.5) THEN
    WRITE(2,5) ANC(M),-ACC(M,1),ACC(M,2),-ACC(M,3)
  ENDIF
60  CONTINUE
  END

```

DATA FOR THE CALCULATION OF THE SATELLITE INFRARED EMISSION

G

Face	Node	Material	Area (m ²)	Emissivity	Normal
S1+	1	SSM	0.514	0.15	z
	2	kapton	7.179	0.45	z
	3	kapton doré	1.308	0.23	z
S1-	4	SSM	0.797	0.15	-z
	5	kapton	3.583	0.45	-z
S2+	6	SSM	1.316	0.15	-x
	7	kapton	5.194	0.45	-x
S2-	8	SSM	1.339	0.15	x
	9	kapton	5.171	0.45	x
S3+	10	SSM	1.050	0.15	-y
	11	kapton	2.465	0.45	-y
S3-	12	kapton	5.806	0.45	ay+bz

with a = 0.60543 and b = -0.79590

37.76

Face	Node	Temperature (°K)					
		0°	90°	180°	SHADIN	SHADOUT	360°
S1+	1	305 + 15 cos(θ-90)					
	2	338		188			
	3	338		188			
S1-	4	282.5 - 7.5 cos(θ-90)					
	5	273					
S2+	6	270 - 20 sin(θ-90)					
	7	188	338			188	
S2-	8	270 + 20 sin(θ-90)					
	9	188	338			188	
S3+	10	275			188	275	
	11	338			188	338	
S3-	12	188					

SHADIN & SHADOUT : angles of entry and exit of the earth's shadow

Theoretical calculation of the force Induced by the solar flux on the solar array

There are 3 components in the resulting force :

- the force created by the direct irradiation

$$F_1 = -\frac{1}{c} GA \cos \theta \mathbf{s}$$

- the force created by diffuse reflexion

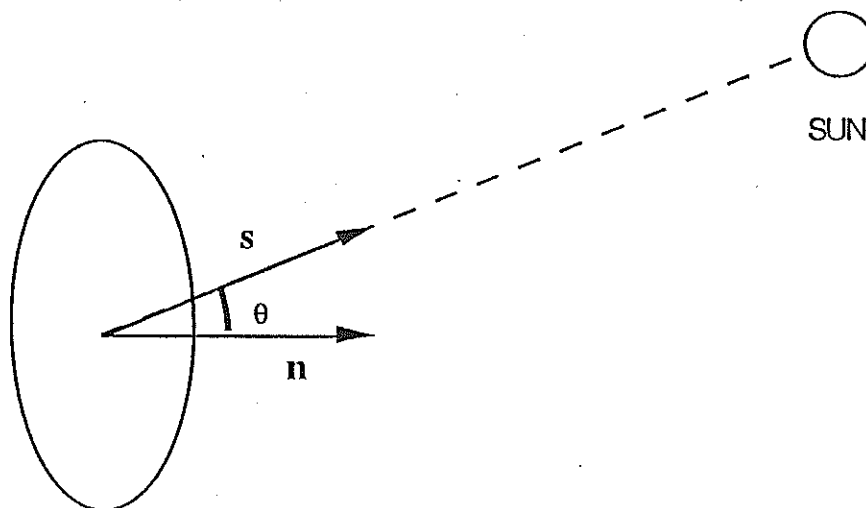
$$F_d = -\frac{1}{c} GA D(f) \rho (1-\beta) \cos \theta \mathbf{n}$$

- the force created by the specular reflexion

$$F_s = -\frac{1}{c} GA \beta \rho \mathbf{n}$$

with the following meanings for symbols

c : speed of light	2.9979 10 ⁸ ms ⁻¹
G : solar flux	1317.1 Wm ⁻²
A : Area of the array	19.503 m ²
ρ : reflexion coefficient	0.31000
β : specular percentage	0.51613
M : satellite weight	1850.0 kg
D(f) : diffuse function	0.66667



\mathbf{s} : directional vector from the solar array towards the sun
 \mathbf{n} : unit normal of the solar array

H.2

In our case, the solar array is normal to the sun, then $\theta = 0$ and $\mathbf{n} = \mathbf{s}$
Therefore :

$$F_i = -\frac{1}{c} GA \mathbf{n}$$

$$F_d = -\frac{1}{c} GA D(f) \rho (1-\beta) \mathbf{n}$$

$$F_s = -\frac{1}{c} GA \beta \rho \mathbf{n}$$

Then, the total force F acting on the solar array is given by :

$$F = F_i + F_d + F_s = -\frac{1}{c} GA (1 + (1-\beta) \rho D(f) + \beta \rho) \mathbf{n}$$

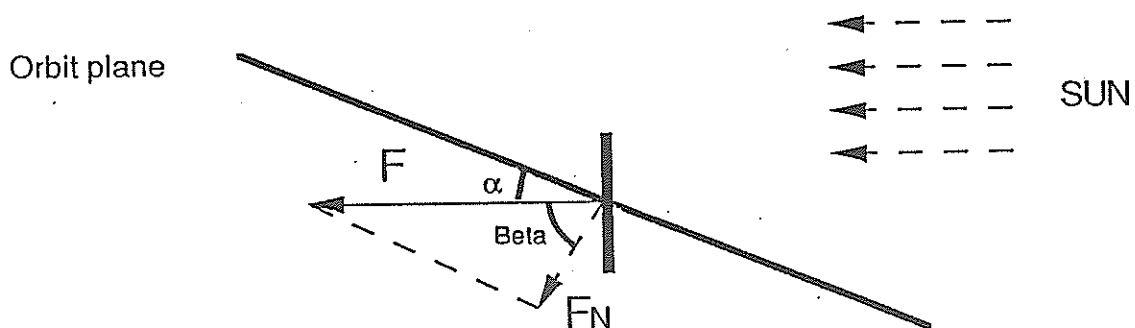
Numerical application : $F = 1.07966 \cdot 10^{-4} \text{ N}$

The acceleration is given by :

$$\gamma = \frac{F}{M}$$

Numerical application : $\gamma = 58.3 \cdot 10^{-9} \text{ ms}^{-2} = 58.3 \text{ nms}^{-2}$

Calculation of the component of the force normal to the orbit plane



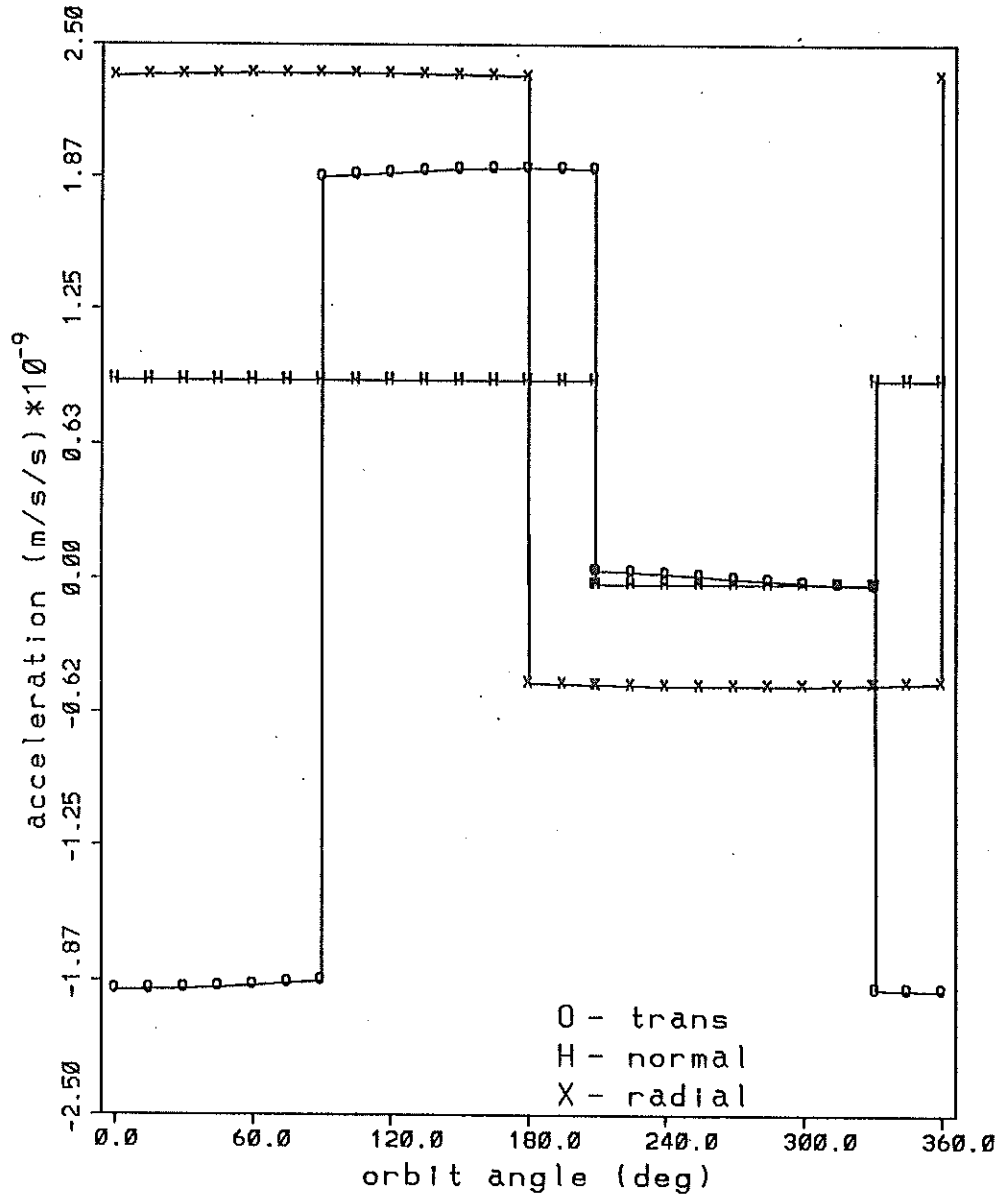
We have :

$$\sin \alpha = \frac{F_N}{F}$$

then $F_N = F \sin \alpha$ with $\alpha = 90 - \beta$
 $\beta = 16.424^\circ$ $\alpha = 73.576^\circ$

$F_N = 3.0527 \cdot 10^{-5} \text{ N}$ $\gamma_N = 19.1064 \text{ nms}^{-2}$

Satellite infrared forces
June 23, 1989



PROGRAM DRAG

J.1

```

*****
This program calculates the factors A*Cx, A*Cy, A*Cz used in the
computation of the atmospheric drag of the SPOT satellite.
(these factors are given in m2)
*****

```

```

REAL TA,RO,W,R,PI,Q,U,ALPHA,S,V,ERF,ER,EX,RT,SPI
REAL SUNANG,CIGMA,SHADIN,SHADOU,H,A1,A2,B1,B2
REAL CLY13,CLZ13,CLY14,CLZ14
INTEGER I,J,K
DIMENSION A(14),T(14,25),CR(14,25),CF(14,25),CLP(14,25,3),O(14,25)
DIMENSION TWR(14,25),TWF(14,25),TG(14,25),SC(3,25),ANC(25)
DIMENSION CDA(14,25),CDR(14,25),CLA(14,25),WMM(25)
DIMENSION CLR(14,25),CD(14,25),CL(14,25),CLR2(14,25)
OPEN(19,STATUS='OLD',FORM='FORMATTED',FILE='drag.tnr')
OPEN(15,STATUS='OLD',FORM='FORMATTED',FILE='masmoy.dat')

```

```

*** Definition of the area of each surface (in m**2) ***

```

```

A(1)=0.514
A(2)=7.179
A(3)=1.308
A(4)=0.797
A(5)=3.583
A(6)=1.316
A(7)=5.194
A(8)=1.339
A(9)=5.171
A(10)=1.050
A(11)=2.465
A(12)=5.086
A(13)=19.503
A(14)=19.503

```

```

*** Definition of constants ***

```

```

W: weight of the satellite (kg)
W=1850.0
R: Boltzmann's gas constant (kg*m**2/mole/sec**2/K)
R=8.32
Q: Constant used in the calculation of the coefficient alpha
Q=3.6
V: Velocity of the satellite (m/s)
V=7440.0
PI: obvious, no.....
PI=3.14159
SUNANG: angle between the periapsis and the orbital noon (deg.)
SUNANG=49.825
CIGMA: angle between the sunrise and the orbital noon (deg.)
CIGMA=90.0
SHADIN: angle of entry in the shade (deg.)
SHADIN=169.00+90.-SUNANG
SHADOU: angle of exit of the shade (deg.)
SHADOU=290.88+90.-SUNANG

```

```

*** Definition of the temperature of each surface ***

```

```

DO 5 I=1,25
  H=(I-1)*15.
  ANC(I)=H
  T(1,I)=305.+15*COS((H-CIGMA)*PI/180.)
  T(4,I)=282.5-7.5*COS((H-CIGMA)*PI/180.)
  T(6,I)=270.-20*SIN((H-CIGMA)*PI/180.)
  T(8,I)=270.+20*SIN((H-CIGMA)*PI/180.)

```

```

T(5,I)=273.
T(12,I)=188.
IF ((H.GT.(CIGMA+90)).AND.(H.LT.(CIGMA+270))) THEN
  T(2,I)=188.
  T(3,I)=188.
ELSE
  T(2,I)=338.
  T(3,I)=338.
ENDIF
IF ((H.GT.CIGMA).AND.(H.LT.SHADOU)) THEN
  T(7,I)=188.
ELSE
  T(7,I)=338.
ENDIF
IF ((H.GT.CIGMA).AND.(H.LT.SHADIN)) THEN
  T(9,I)=338.
ELSE
  T(9,I)=188.
ENDIF
IF ((H.GT.SHADIN).AND.(H.LT.SHADOU)) THEN
  T(10,I)=188.
  T(11,I)=188.
  T(13,I)=213.
  T(14,I)=213.
ELSE
  T(10,I)=275.
  T(11,I)=338.
  T(13,I)=328.
  T(14,I)=328.
ENDIF
CONTINUE
5
C
C   *** Check module of the temperatures ***
C
C   DO 75 NA=1,14
C     WRITE(*,*) 'surface',NA
C     DO 77 NB=1,25
C       WRITE(*,*) 'T(',ANC(NB),')=',T(NA,NB)
C77   CONTINUE
C75  CONTINUE
C
C   *** Definition of the mean molecular weight along the orbit ***
C
C   DO 23 J=1,24
C     READ(15,*) A1,B1
27   READ(15,*) A2,B2
C     IF (A2.GT.ANC(J)) THEN
C       WMM(J)=B1+(((B2-B1)*(ANC(J)-A1))/(A2-A1))
C     ELSE
C       A1=A2
C       B1=B2
C       GOTO 27
C     ENDIF
23   CONTINUE
C     WMM(25)=WMM(1)
C
C   *** Definition of the temperature Tw=CF*TWF+CR*TWR ***
C
C   DO 30 I=1,14
C     DO 32 J=1,25
C       CF(I,J)=0.
C       CR(I,J)=0.
32   CONTINUE
30  CONTINUE
DO 34 J=1,25
  CF(6,J)=1.0

```

```

CF (7, J)=1.0
CR (8, J)=1.0
CR (9, J)=1.0
IF ((ANC (J) .GT. 90.) .AND. (ANC (J) .LT. 270.)) THEN
  CR (13, J)=1.0
  CF (14, J)=1.0
ELSE
  CF (13, J)=1.0
  CR (14, J)=1.0
ENDIF
34 CONTINUE
C
C *** Definition of the incidence of each surface ***
C
DO 40 J=1, 25
  O (1, J)=0.
  O (2, J)=0.
  O (3, J)=0.
  O (4, J)=0.
  O (5, J)=0.
  O (6, J)=PI/2.
  O (7, J)=PI/2.
  O (8, J)=1.5*PI
  O (9, J)=1.5*PI
  O (10, J)=0.
  O (11, J)=0.
  O (12, J)=0.
  O (13, J)=ASIN (COS (19.5* (PI/180.)) *COS (ANC (J) * (PI/180.)))
  O (14, J)=O (13, J)+PI
40 CONTINUE
C
C *** Check module of the incidence of each surface ***
C
DO 45 I=1, 14
  DO 47 J=1, 25
    WRITE (*, *) 'O (' , I , ' , ' , ANC (J) , ' ) = ' , O (I, J)
C47 CONTINUE
C45 CONTINUE
C
C *** Definition of several parameters ***
C
TA=1200.
S=V/SQRT (2*R*TA)
SPI=SQRT (PI)
C
C *** Calculation of the drag & lift coefficients ***
C
DO 50 I=1, 14
  DO 55 J=1, 25
    IF (WMM (J) .LT. 16.) THEN
      U=WMM (J) /16.
    ELSE
      U=1.0
    ENDIF
    ALPHA=(Q*U) / ((1+U)**2)
C
C TWF : Temperature of the front surface
C *****
C TWF (I, J)=(SQRT (T (I, J) ) + ((SQRT ((V**2) / (2*R) ) -SQRT (T (I, J) ) )
+ *SQRT (1-ALPHA) ) ) **2
C
C TWR : Temperature of the rear surface
C *****
C TWR (I, J)=(SQRT (T (I, J) ) + ((SQRT (TA) -SQRT (T (I, J) ) )
+ *SQRT (1-ALPHA) ) ) **2
C

```



```

C      TG : Temperature of the surface (either TWF or TWR )
C      *****
C      TG(I,J)=(CF(I,J)*TWF(I,J)+(CR(I,J)*TWR(I,J))

C      CALL FUNC(S*SIN(O(I,J)),ERF)
C      ER=1+ERF
C      EX=EXP(-(S*SIN(O(I,J)))**2))
C      RT=SQRT(TG(I,J)/TA)

C      CLR : Lift coefficient due to the reemission
C      *****
C      CLR(I,J)=ABS(((SPI*COS(O(I,J))*SIN(O(I,J))*RT*ER)/(2*S))
+      +(COS(O(I,J))*RT*EX)/(2*S*S))

C      CDR : Drag coefficient due to the reemission
C      *****
C      CDR(I,J)=((SPI*((SIN(O(I,J)))**2)*RT*ER)/(2*S))
+      +((SIN(O(I,J))*RT*EX)/(2*S*S))

C      CLA : Lift coefficient due to the absorption
C      *****
C      CLA(I,J)=(COS(O(I,J))*ER)/(2*S*S)

C      CDA : Drag coefficient due to the absorption
C      *****
C      CDA(I,J)=(SIN(O(I,J))*ER*(1+(1/(2*S*S))))+(EX/(SPI*S))

C      Total coefficients of drag & lift (absorption + reemission)
C      *****
C      CD(I,J)=CDR(I,J)+CDA(I,J)
C      CL(I,J)=CLR(I,J)+CLA(I,J)
C      *****

C      *** Check module of the lift & drag coefficient ***

C      WRITE(*,*) ' '
C      WRITE(*,*) '*** CD & CL Coefficients ***'
C      WRITE(*,*) ' '
C      WRITE(*,*) 'CDA(' ,I,' , , ,ANC(J),')=' ,CDA(I,J)
C      WRITE(*,*) 'CDR(' ,I,' , , ,ANC(J),')=' ,CDR(I,J)
C      WRITE(*,*) 'CLA(' ,I,' , , ,ANC(J),')=' ,CLA(I,J)
C      WRITE(*,*) 'CLR(' ,I,' , , ,ANC(J),')=' ,CLR(I,J)
C      WRITE(*,*) '$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$ '
C      WRITE(*,*) 'CD(' ,I,' , , ,ANC(J),')=' ,CD(I,J)
C      WRITE(*,*) 'CL(' ,I,' , , ,ANC(J),')=' ,CL(I,J)
C      WRITE(*,*) '$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$ '
55      CONTINUE
50      CONTINUE

C      *** Calculation of the lift components along the Y&Z axis ***

C      DO 10 I=1,14
C          DO 15 J=1,25
C              DO 20 K=1,3
C                  CLP(I,J,K)=0.
20          CONTINUE
15          CONTINUE
10          CONTINUE
          DO 22 J=1,25
              IF (SIN(ANC(J)).EQ.0.) THEN
                  CLP(13,J,3)=0.
              ELSE
                  CLP(13,J,3)=1./SQRT(1+((TAN(19.5*(PI/180.)))/
+                  SIN(ANC(J)*(PI/180.))**2))
              ENDIF

```


10

```
      BI=Y*I
      BS=Y*(I+1)
      Q=EXP(-(BI**2))+EXP(-(BS**2))
      ERF=ERF+(Q*Y)/2.
CONTINUE
      ERF=ERF*(2./SQRT(PI))
ELSE
      ERF=1.0
ENDIF
IF (X.EQ.0.) THEN
      ERF=0.
ELSE
      ERF=ERF*(Z/X)
ENDIF
RETURN
END
```

DATA FOR THE CALCULATION OF THE ATMOSPHERIC DRAG
OF THE SATELLITE SPOT

K.1

Characteristics of each node :

Face	Node	Material	Area (m ²)	Velocity Incidence (rad.)
S1+	1	SSM	0.514	0.
	2	kapton	7.179	0.
	3	kapton doré	1.308	0.
S1-	4	SSM	0.797	0.
	5	kapton	3.583	0.
S2+	6	SSM	1.316	$\pi/2$
	7	kapton	5.194	$\pi/2$
S2-	8	SSM	1.339	$3\pi/2$
	9	kapton	5.171	$3\pi/2$
S3+	10	SSM	1.050	0.
	11	kapton	2.465	0.
S3-	12	kapton	5.806	0.
SP+	13	cells	19.503	$\arcsin(\cos(v)*\cos(19.5))$
SP-	14	cells	19.503	$\pi+\arcsin(\cos(v)*\cos(19.5))$

with v : true anomaly

DATA FOR THE CALCULATION OF THE ATMOSPHERIC DRAG
OF THE SATELLITE SPOT

K.2

Temperature of each node :

Face	Node	Temperature (°K)					
		0°	90°	180°	SHADIN	SHADOUT	360°
S1+	1	$305 + 15 \cos(\theta-90)$					
	2	338		188			
	3	338		188			
S1-	4	$282.5 - 7.5 \cos(\theta-90)$					
	5	273					
S2+	6	$270 - 20 \sin(\theta-90)$					
	7	188	338			188	
S2-	8	$270 + 20 \sin(\theta-90)$					
	9	188	338			188	
S3+	10	275			188	275	
	11	338			188	338	
S3-	12	188					
SP+	13	328			213	328	
SP-	14	328			213	328	

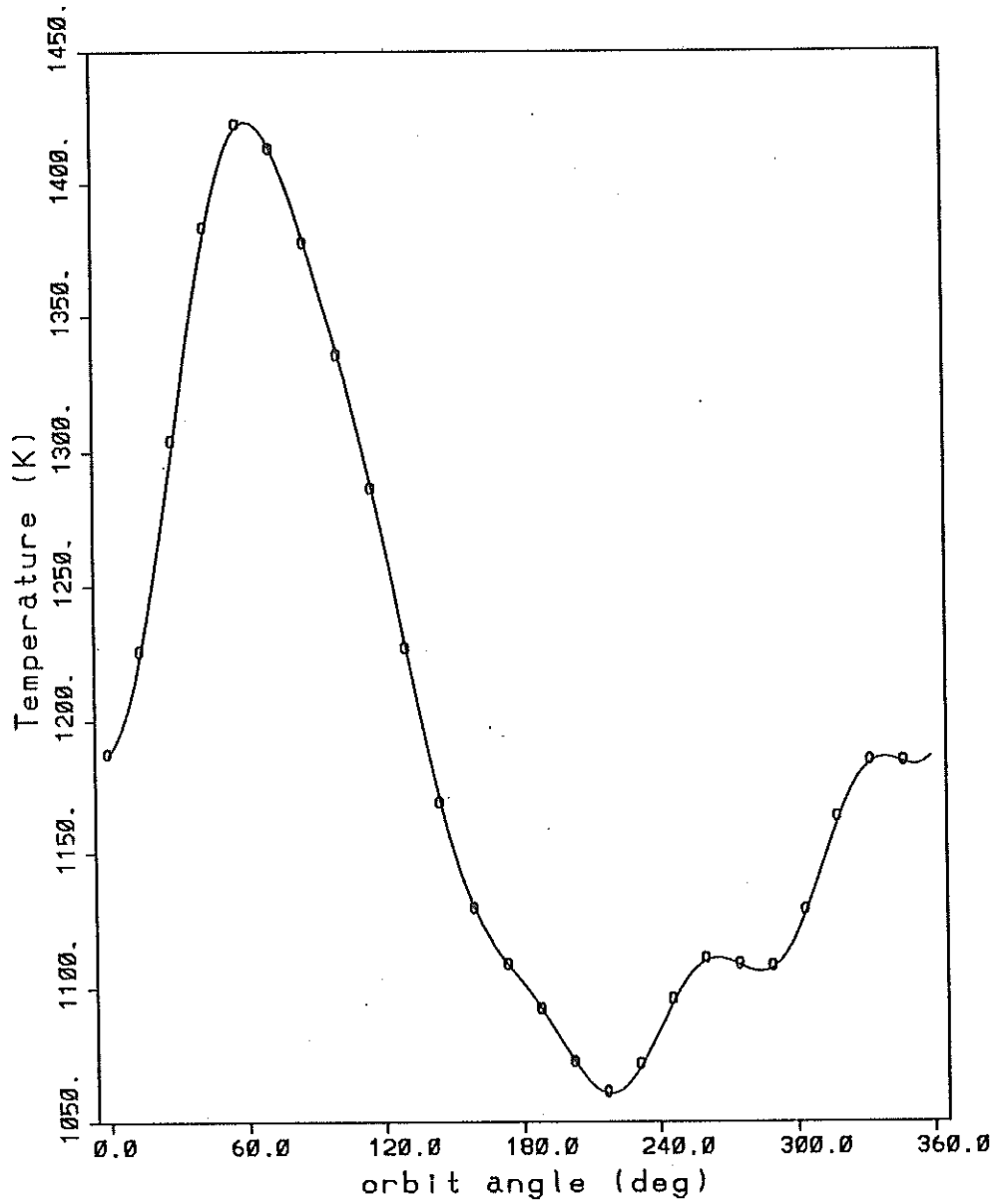
SHADIN & SHADOUT : angles of entry and exit of the earth's shadow

```

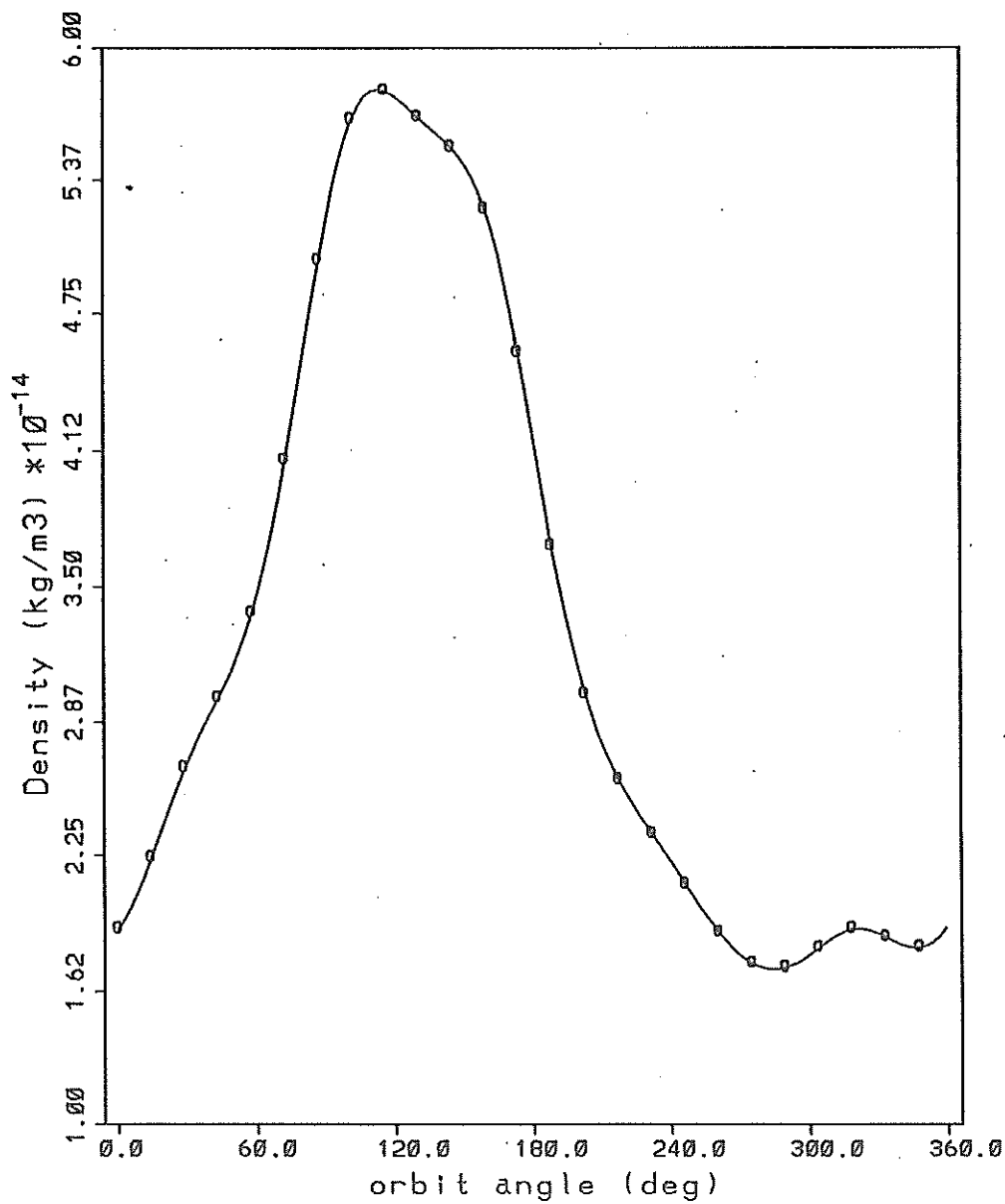
PROGRAM MASMOY
C
C *****
C This program calculates the mean molecular weight of the
C molecules absorbed by the satellite and creates files
C for the densities and temperature.
C *****
C
REAL D, T, ROT, ANG, WMM
INTEGER I
DIMENSION RO(5)
OPEN(1, STATUS='OLD', FORM='FORMATTED', FILE='densite')
OPEN(2, STATUS='OLD', FORM='FORMATTED', FILE='masmoy.dat')
OPEN(3, STATUS='OLD', FORM='FORMATTED', FILE='temp.dat')
OPEN(4, STATUS='OLD', FORM='FORMATTED', FILE='dens5.dat')
OPEN(5, STATUS='OLD', FORM='FORMATTED', FILE='denstot.dat')
REWIND 1
10 READ(1, *, END=20) D, T, RO(1), RO(2), RO(3), RO(4), RO(5), ROT
ANG=(D/101.433)*360.
8 FORMAT( F10.1, G17.7)
9 FORMAT( F10.1, 5G17.7)
WMM=(2*RO(1)+4*RO(2)+16*RO(3)+28*RO(4)+32*RO(5))/ROT
WRITE(2, 8) ANG, WMM
WRITE(3, 8) ANG, T
WRITE(4, 9) ANG, (RO(I), I=1, 5)
WRITE(5, 8) ANG, ROT
GO TO 10
20 CONTINUE
END

```

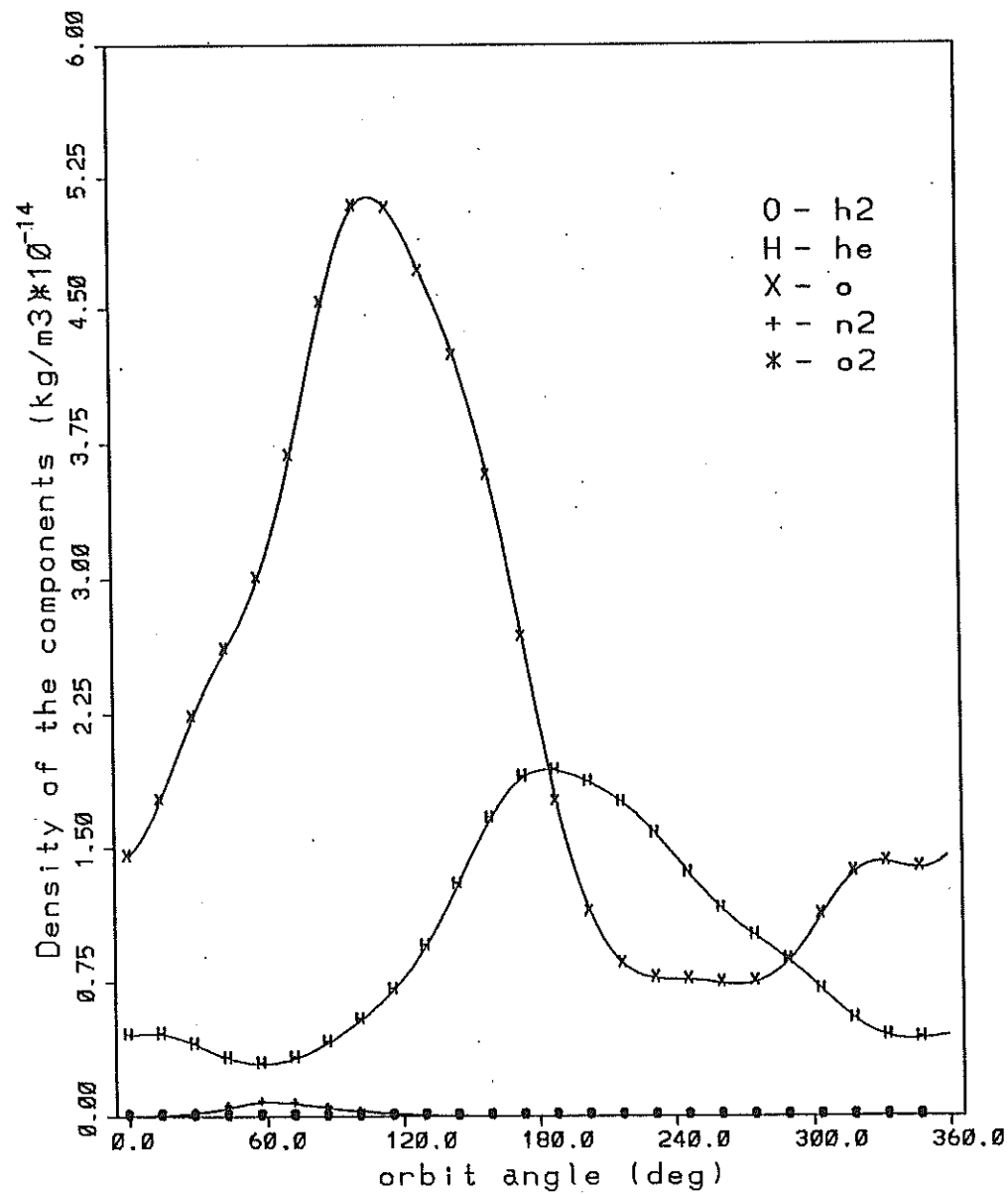
Absolute temperature along one orbit of spot June 23, 1989
(atmospheric model: dtm)



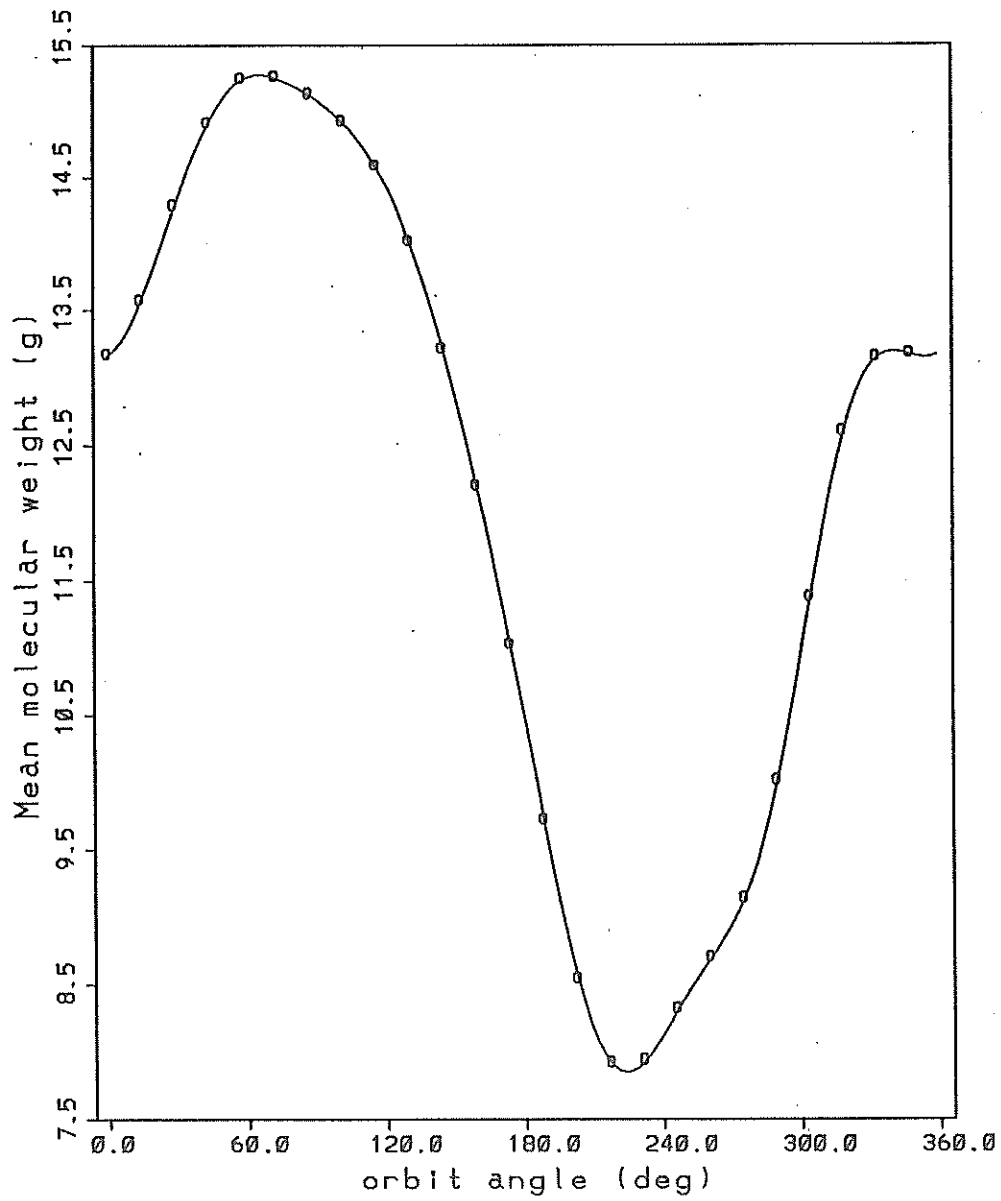
Atmospheric density along one orbit of spot June 23, 1989
(atmospheric model: dtm)



Density of the main components along one orbit of spot June 23, 1989
(atmospheric model: dtm)

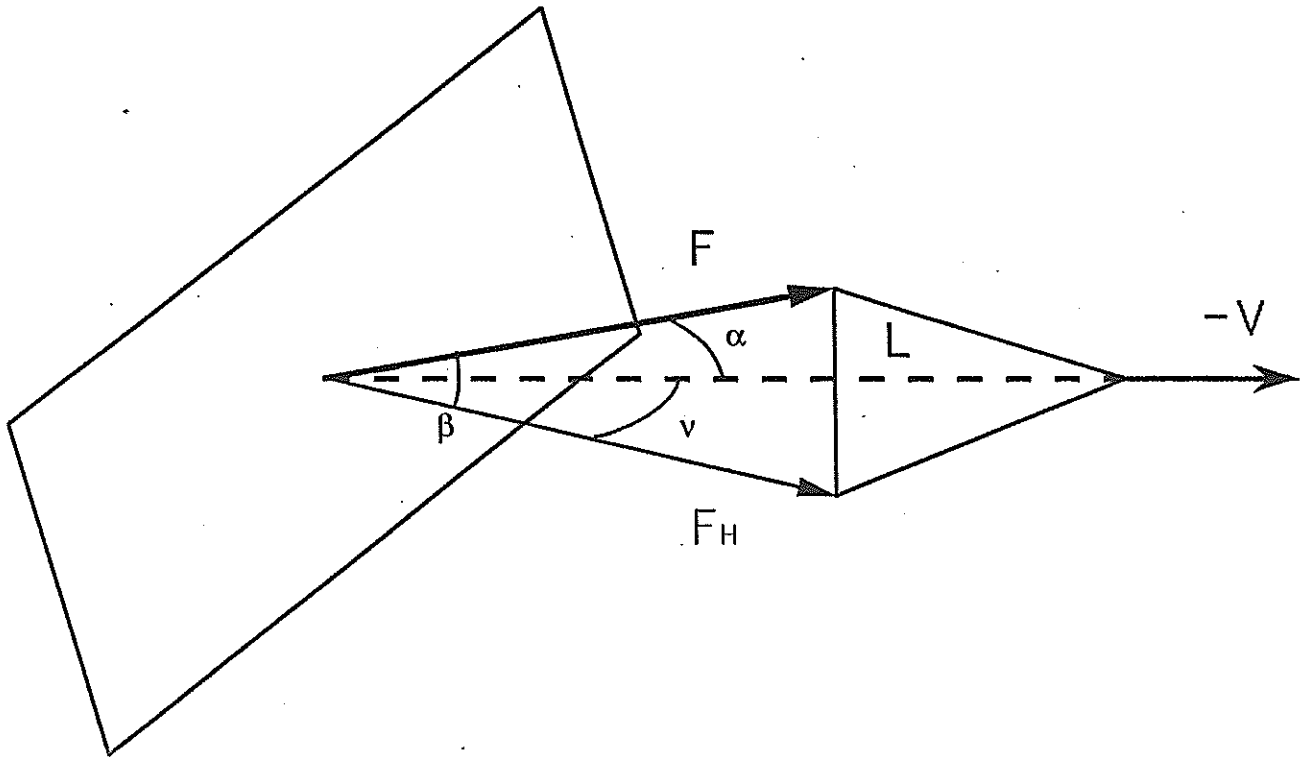


Mean molecular weight along one orbit of spot June 23, 1989
(atmospheric model: dtm)



Calculation of the velocity incidence angle

N



with the following meanings for the symbols :

β : solar array inclination (19.5°)

v : true anomaly

α : $\alpha = 90^\circ - \theta$

θ : velocity incidence angle

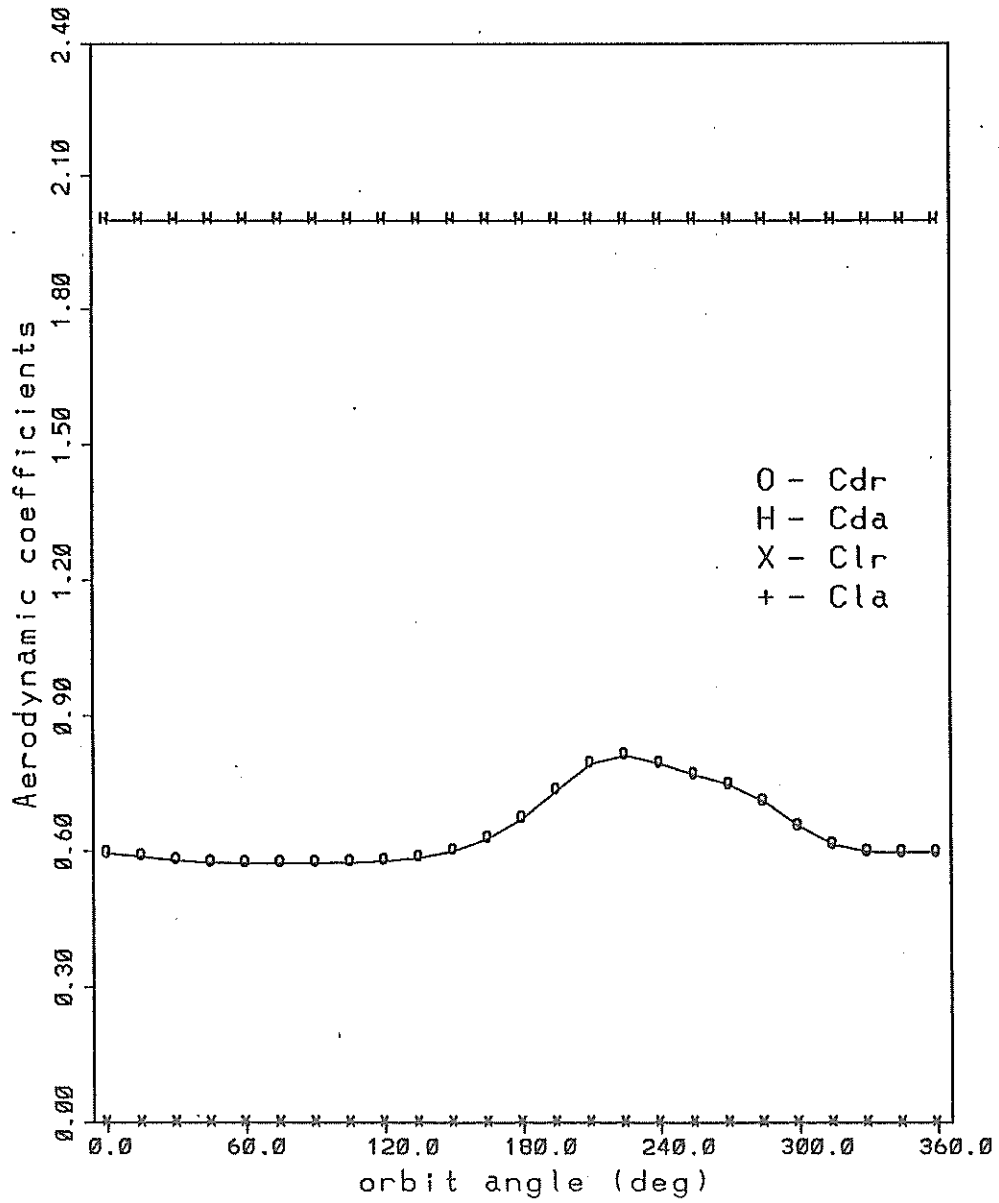
We have $\cos \beta = \frac{F_H}{F}$ $\cos v = \frac{L}{F_H}$ $\cos \alpha = \frac{L}{F}$

Therefore $\cos \alpha = \cos \beta \cos v$

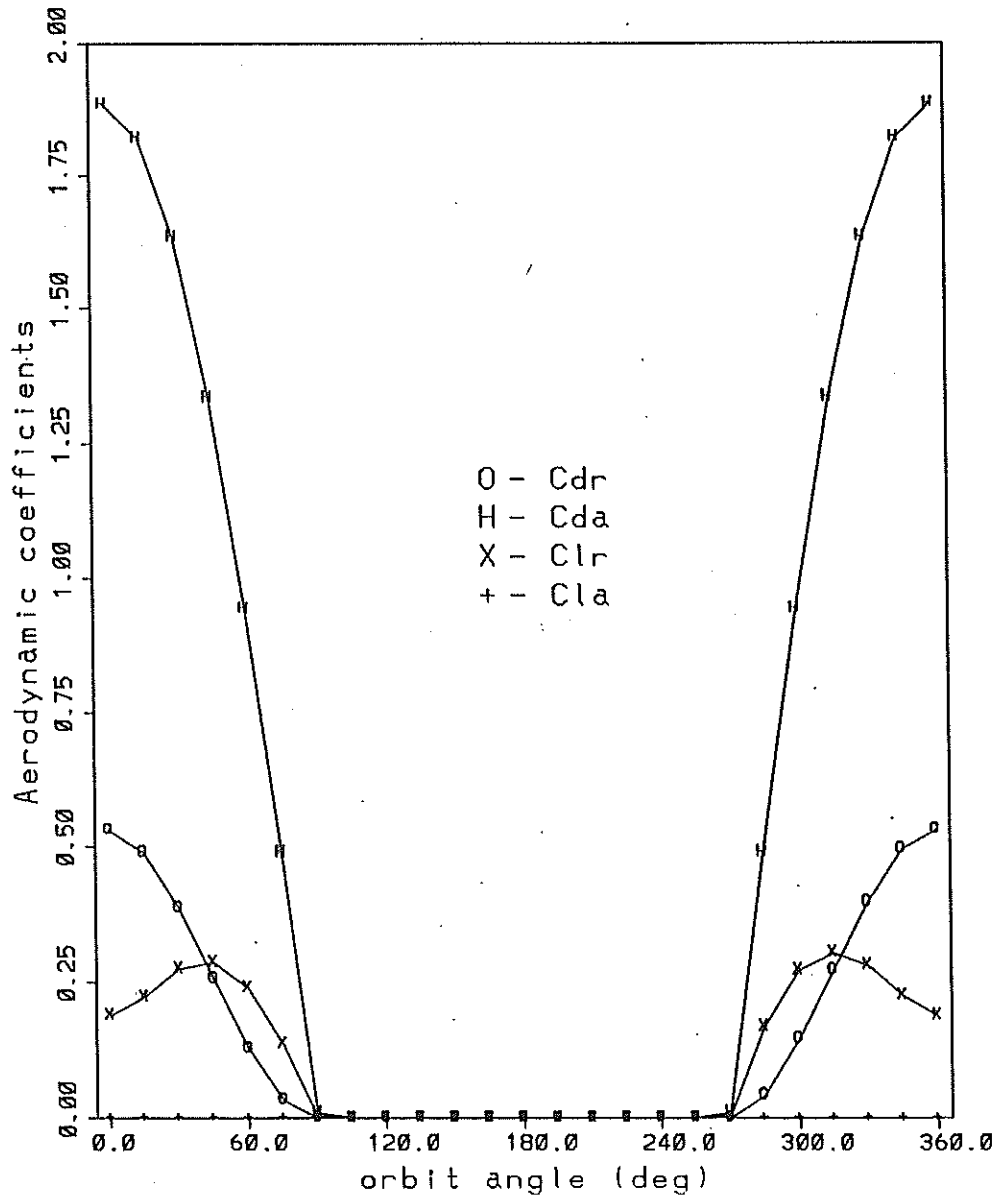
As $\cos \alpha = \cos(90 - \theta) = \sin \theta$

we have finally $\sin \theta = \cos \beta \cos v$

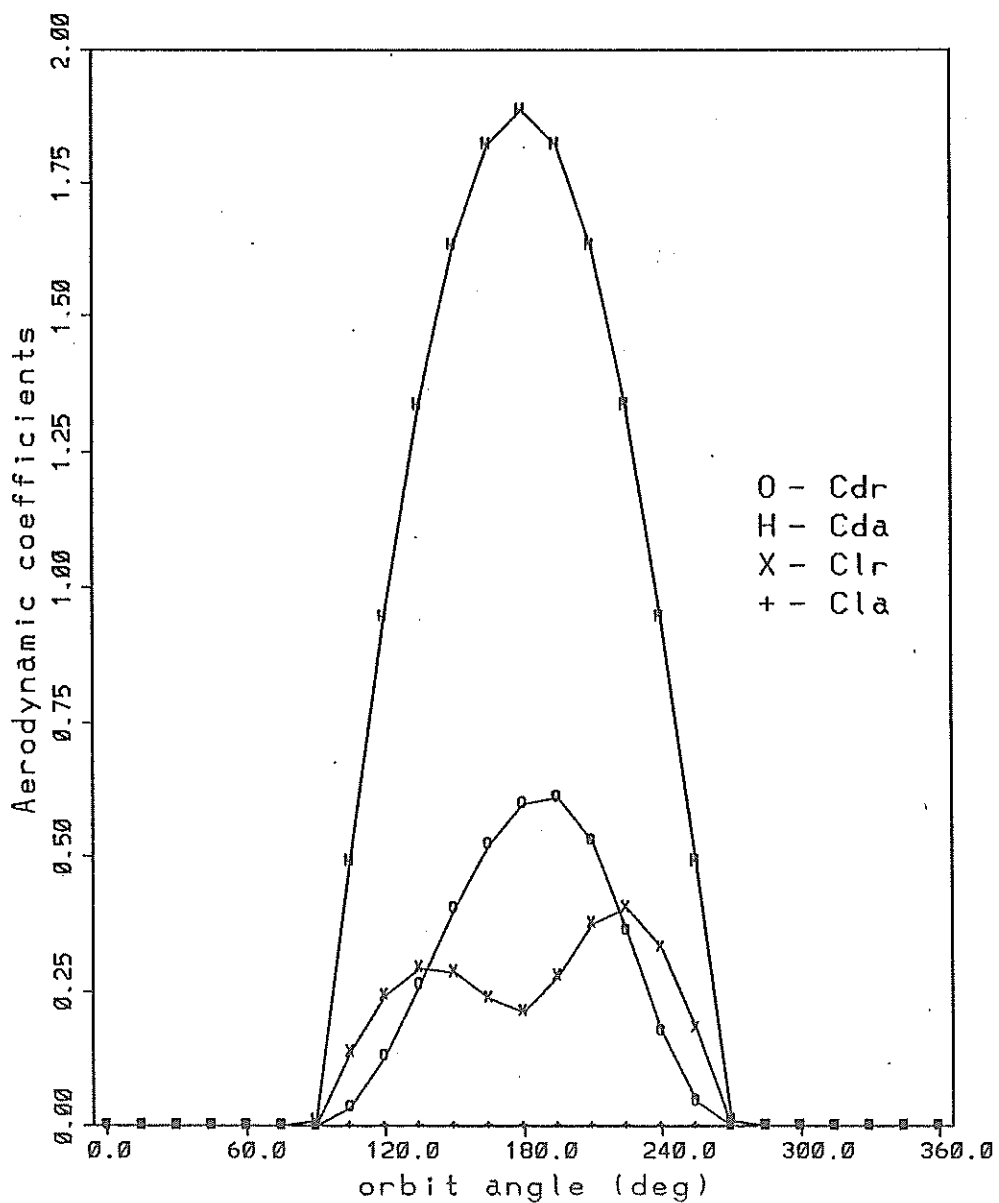
Aerodynamic coefficients along one orbit for face 6



Aerodynamic coefficients along one orbit for face 13.

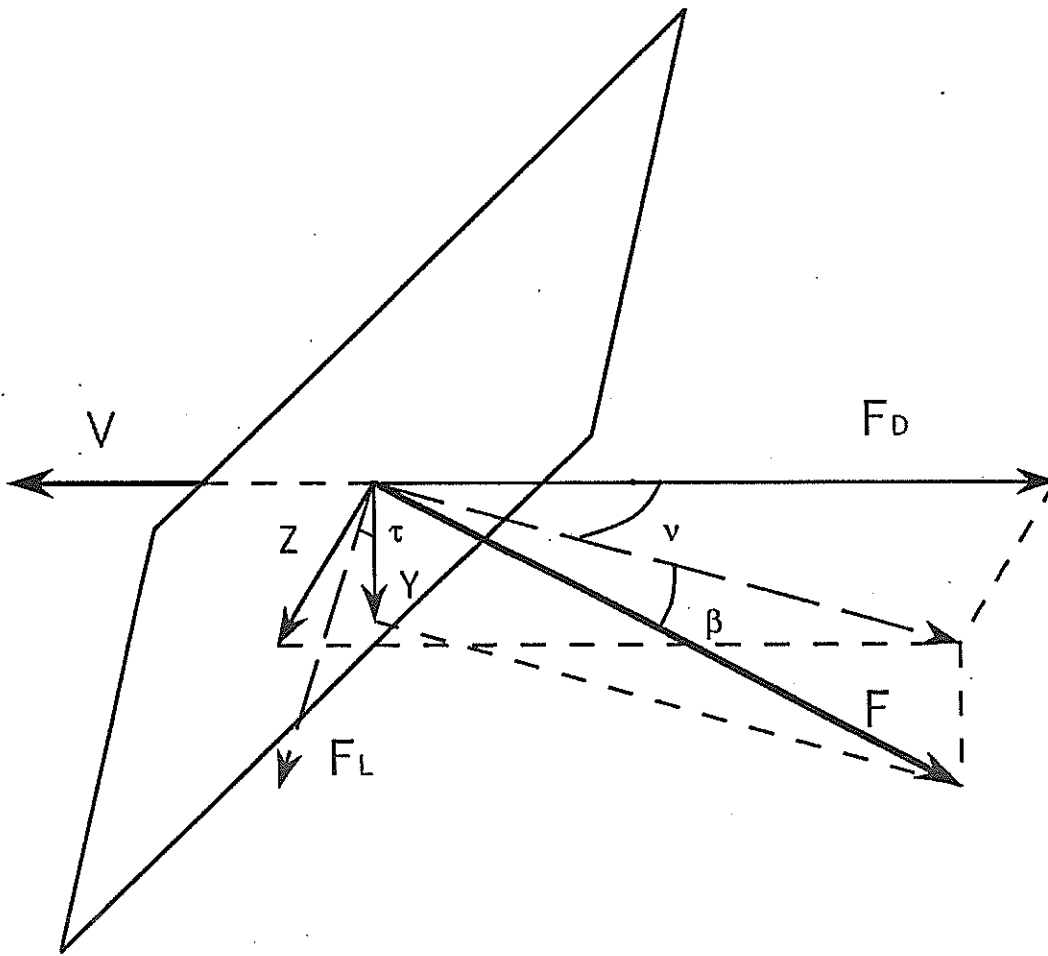


Aerodynamic coefficients along one orbit for face 14



Calculation of the angle tau

Q



with the following meanings for the symbols :

β : solar array inclination (19.5°)

v : true anomaly

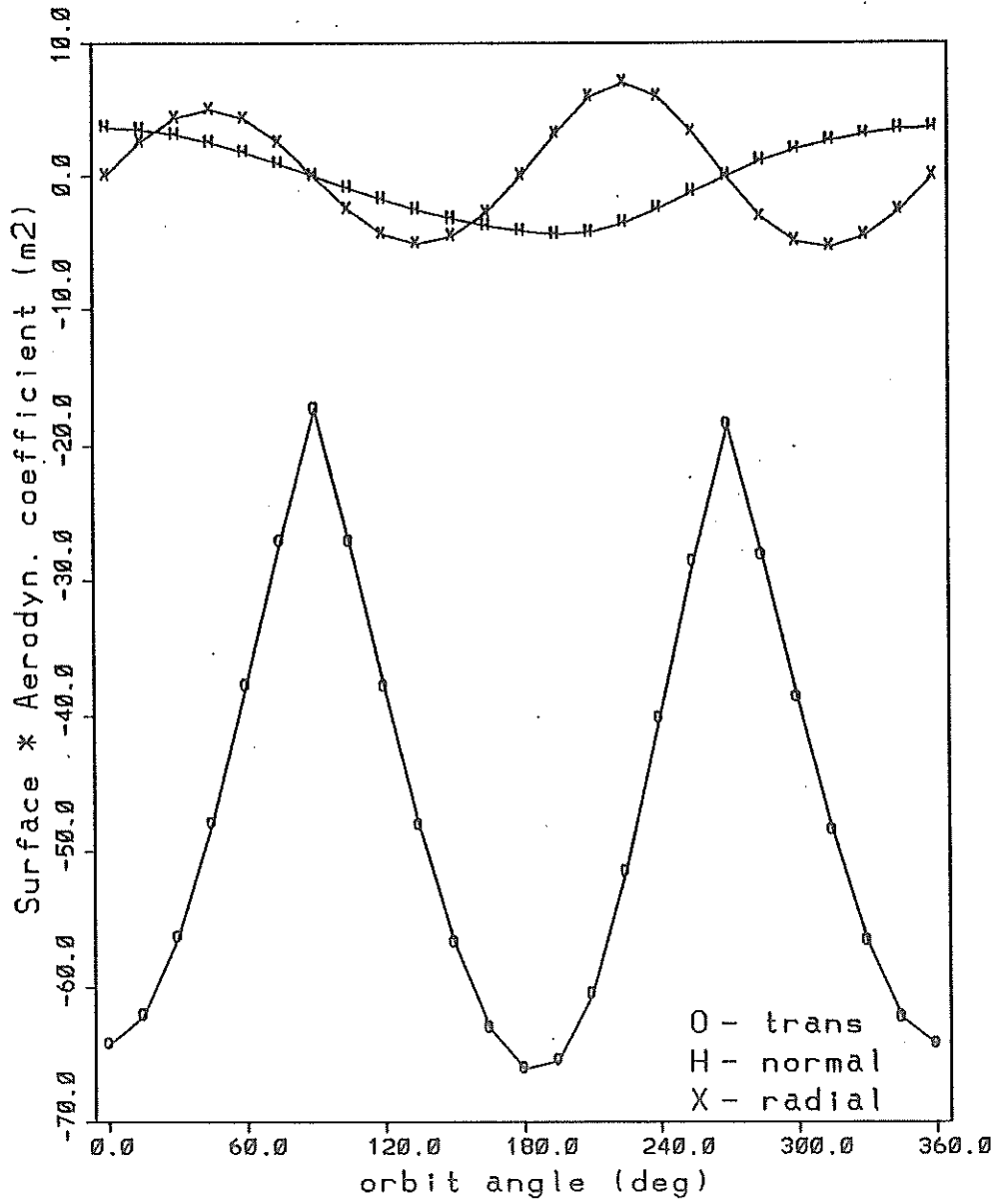
τ : angle between Y and F_L

We have $\tan \tau = \frac{Z}{Y}$ and $\sin v = \frac{Z}{F_H}$ $\tan \beta = \frac{Y}{F_H}$

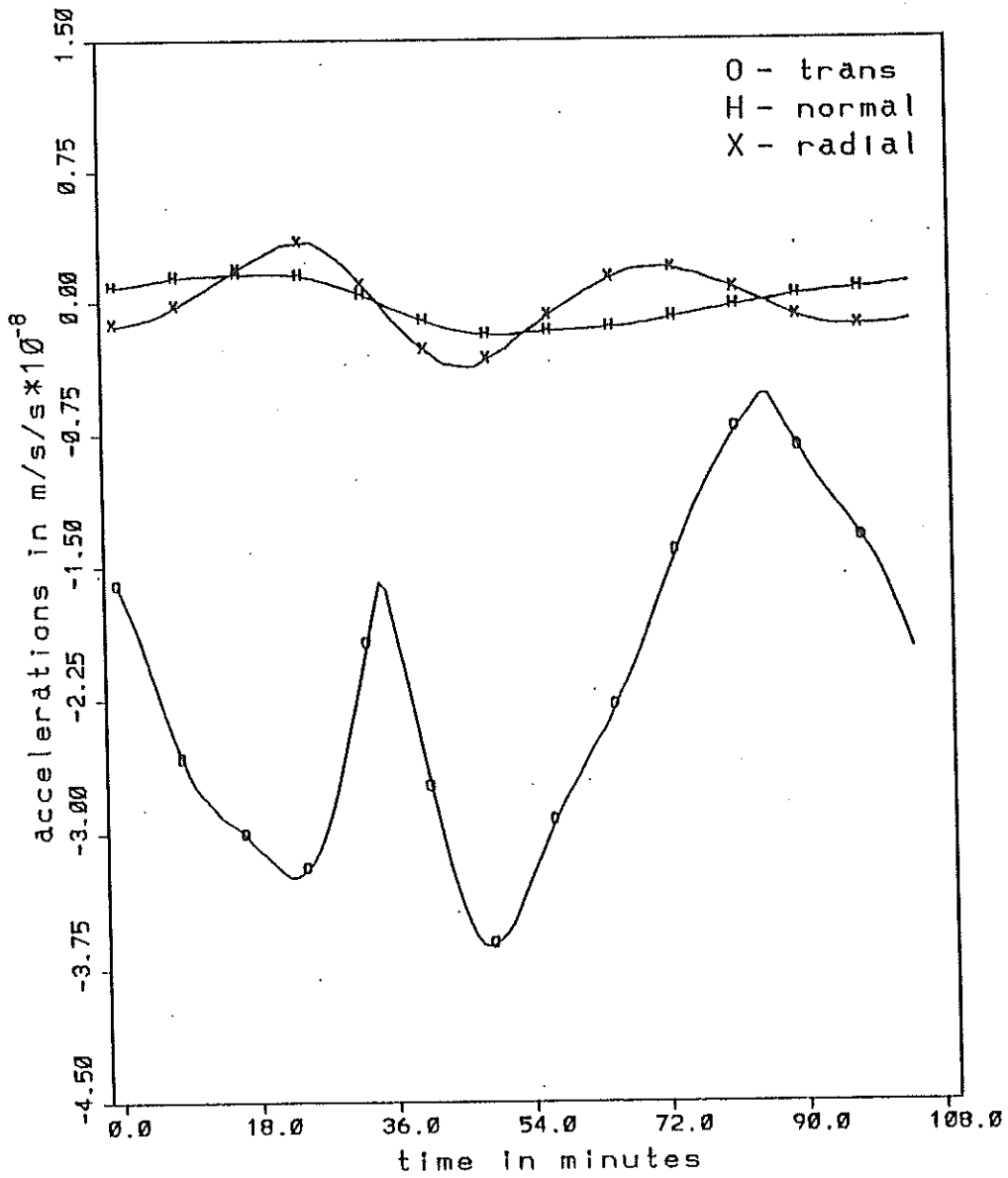
Therefore $\tan \tau = \frac{\sin v}{\tan \beta}$

then $\cos \tau = \frac{1}{\sqrt{1 + \left(\frac{\tan \beta}{\sin v}\right)^2}}$ $\sin \tau = \sqrt{1 - \cos^2 \tau}$

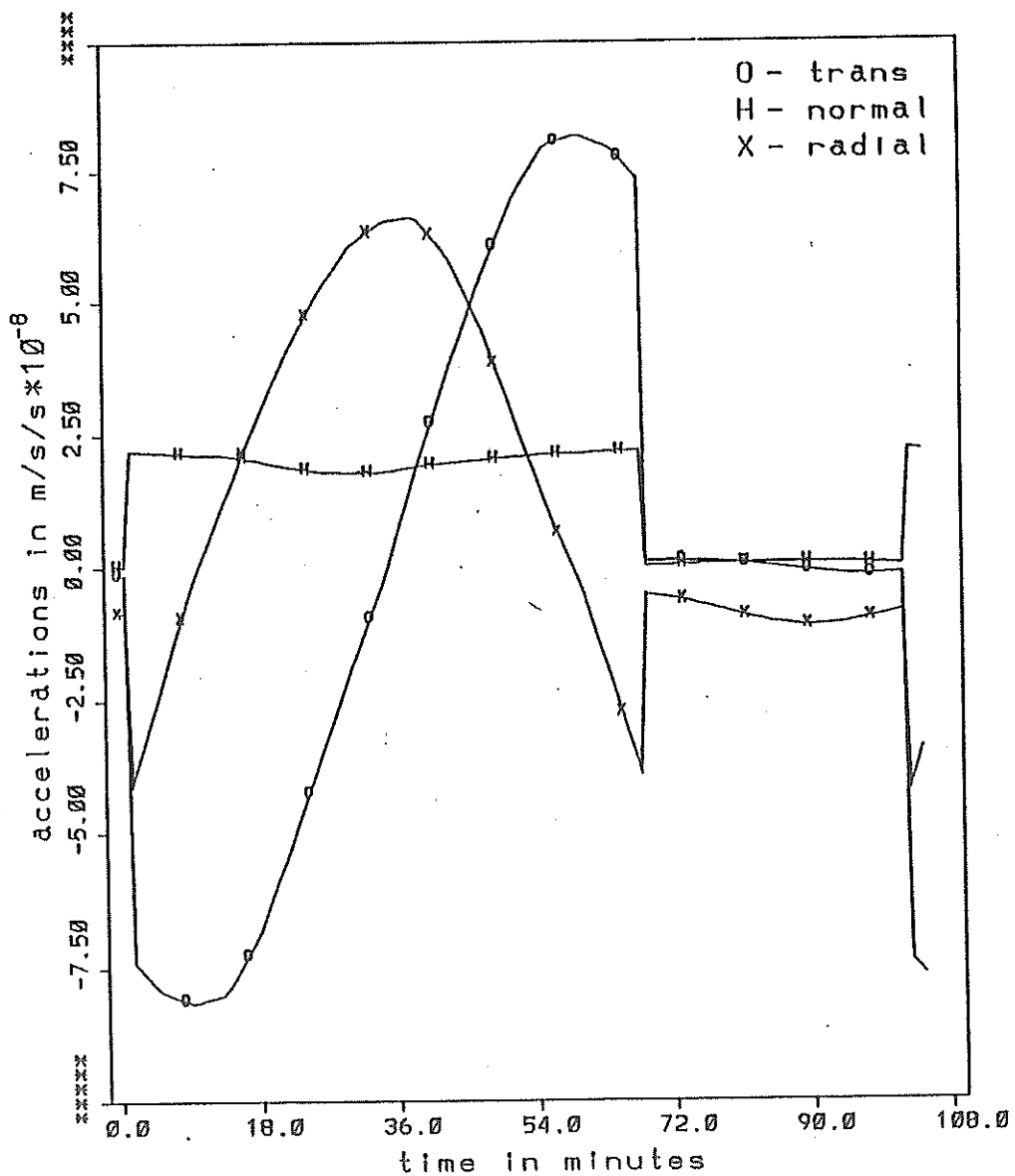
S*C factor
June 23, 1989



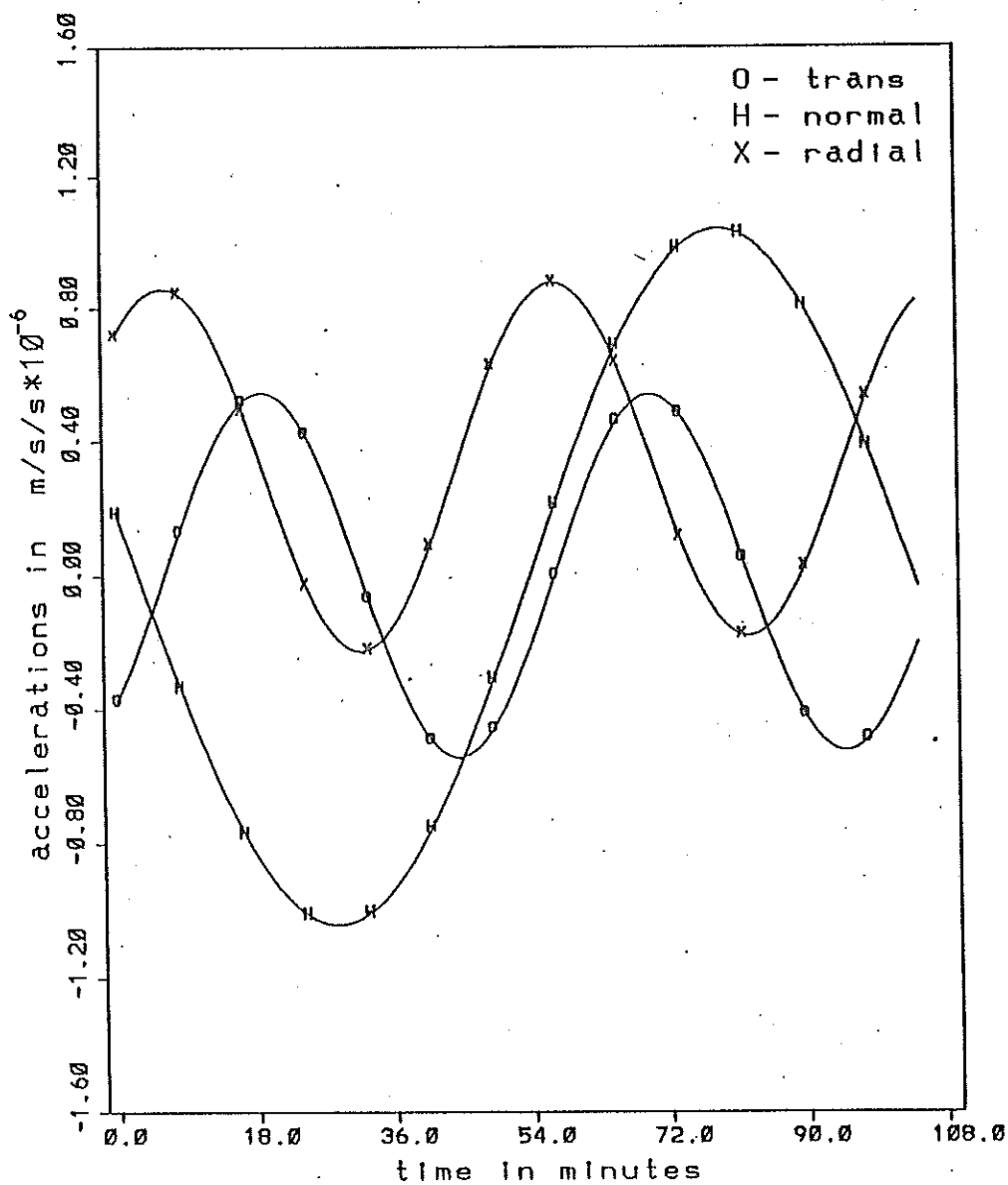
SPOT June 23, 1989
drag acceleration atmospheric model dtm



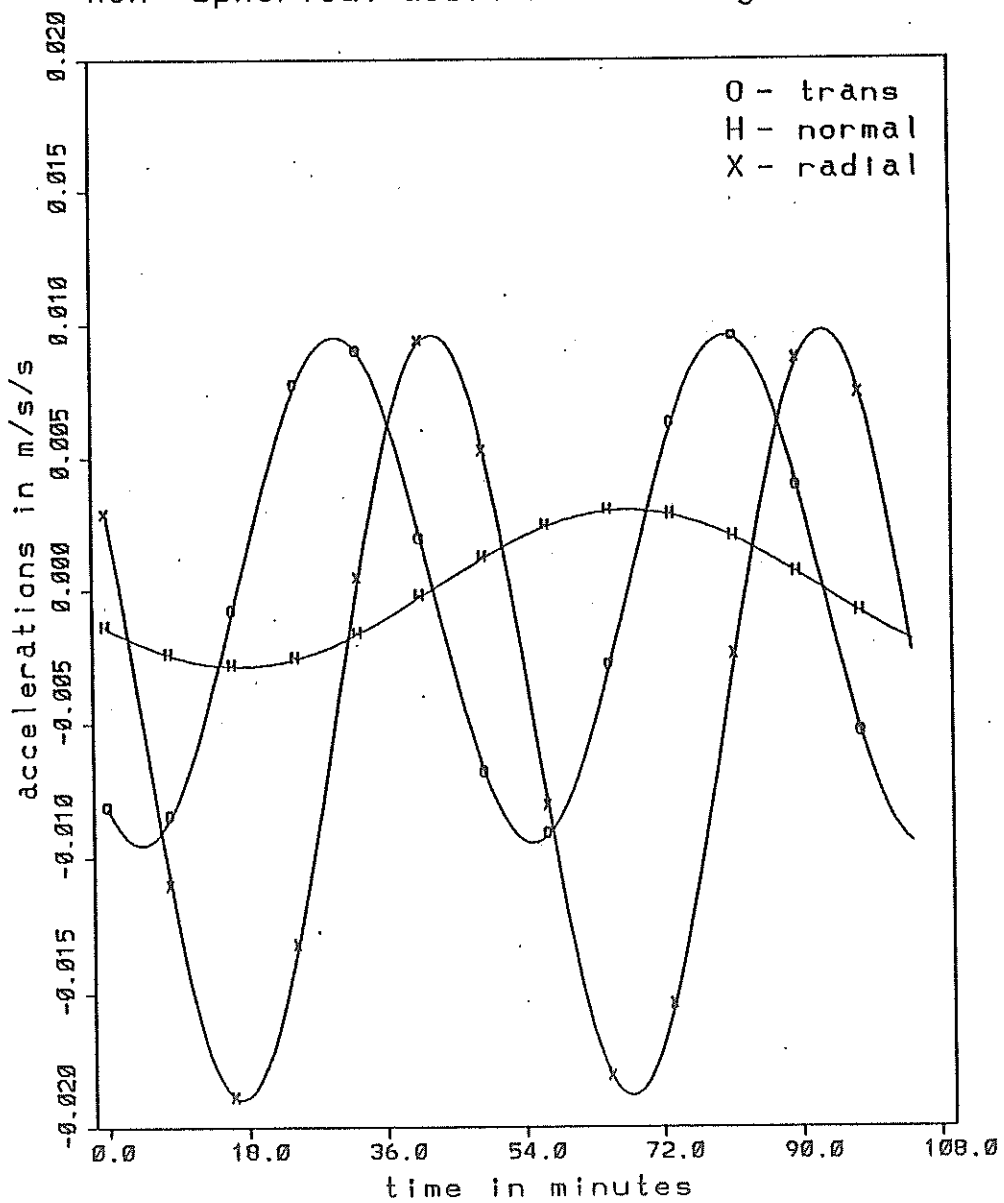
SPOT June 23, 1989
thermal accelerations (solar - albedo - planetary - satellite)



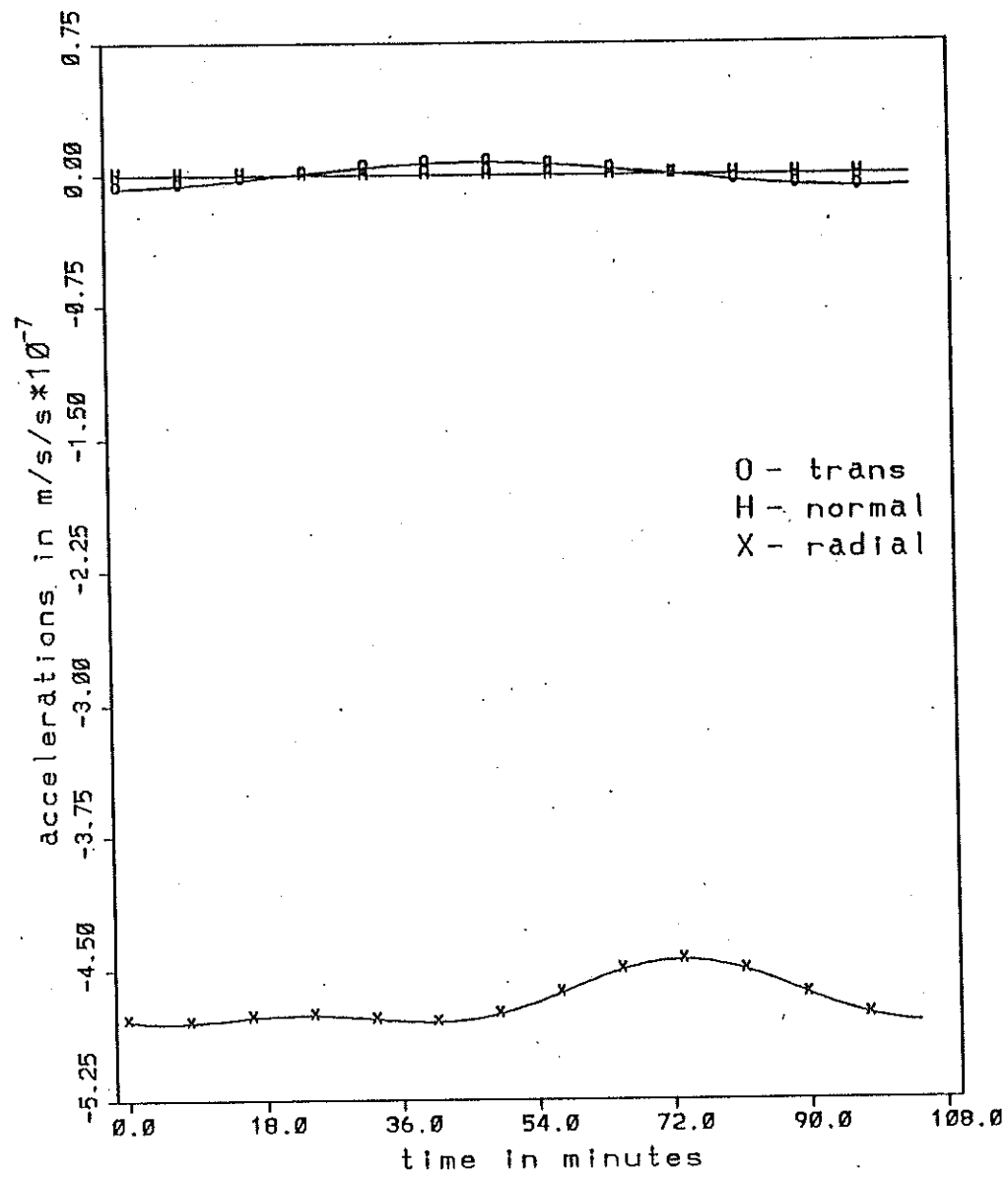
SPOT June 23, 1989
n-body accelerations (sun & moon)



SPOT June 23, 1989
non-spherical accelerations (gemt2 36*36)



SPOT June 23, 1989
relativistic accelerations



spot orbit simulation - une orbite -
 june 23,1989 - toutes les forces - cdrag & tots.tnr

```

initial
  epoch1          1989.0          6.0          23.0
  epoch2          5.              0.           24.
  pos    1 1      7205000.0        1.51d-3      98.7
  vel                    102.50          249.70      287.0
  end

runmode
  mode    2
  iter    0
  end

forces
  satid          8600101.0          1850.0
  sun           1
  drag          5          6.51385          2.70          1.0
  dtides        1
  etper         3
  polmot        1
  relprt        2
  spdut1        1
  geo           1 0
  gm            398600.436
  moon          1
  mmax          36
  njmax         36
  nmax          36
  exfrot        3
  extrad        1
  shadow        0          6378137.0
  end

integ/out
  tstart          0.
  tf              6086.
  nprint         5
  dtnew          17.0
  end

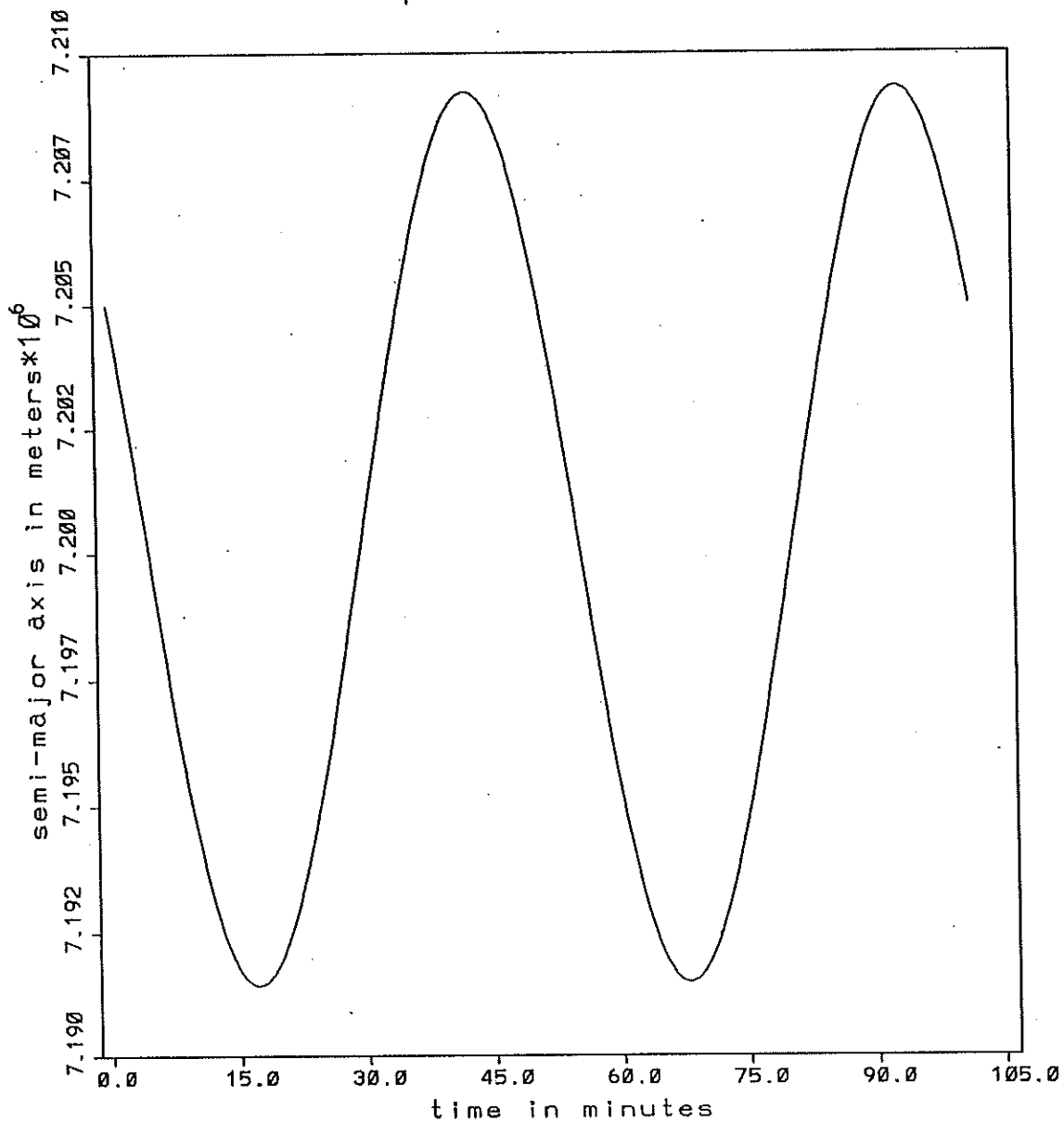
sta/obs
  chordp
  measx         1
  end

files
  utcwrst       1          17.
  utcwrst       2          17.
  densit        0
  accsat        0 0
  end

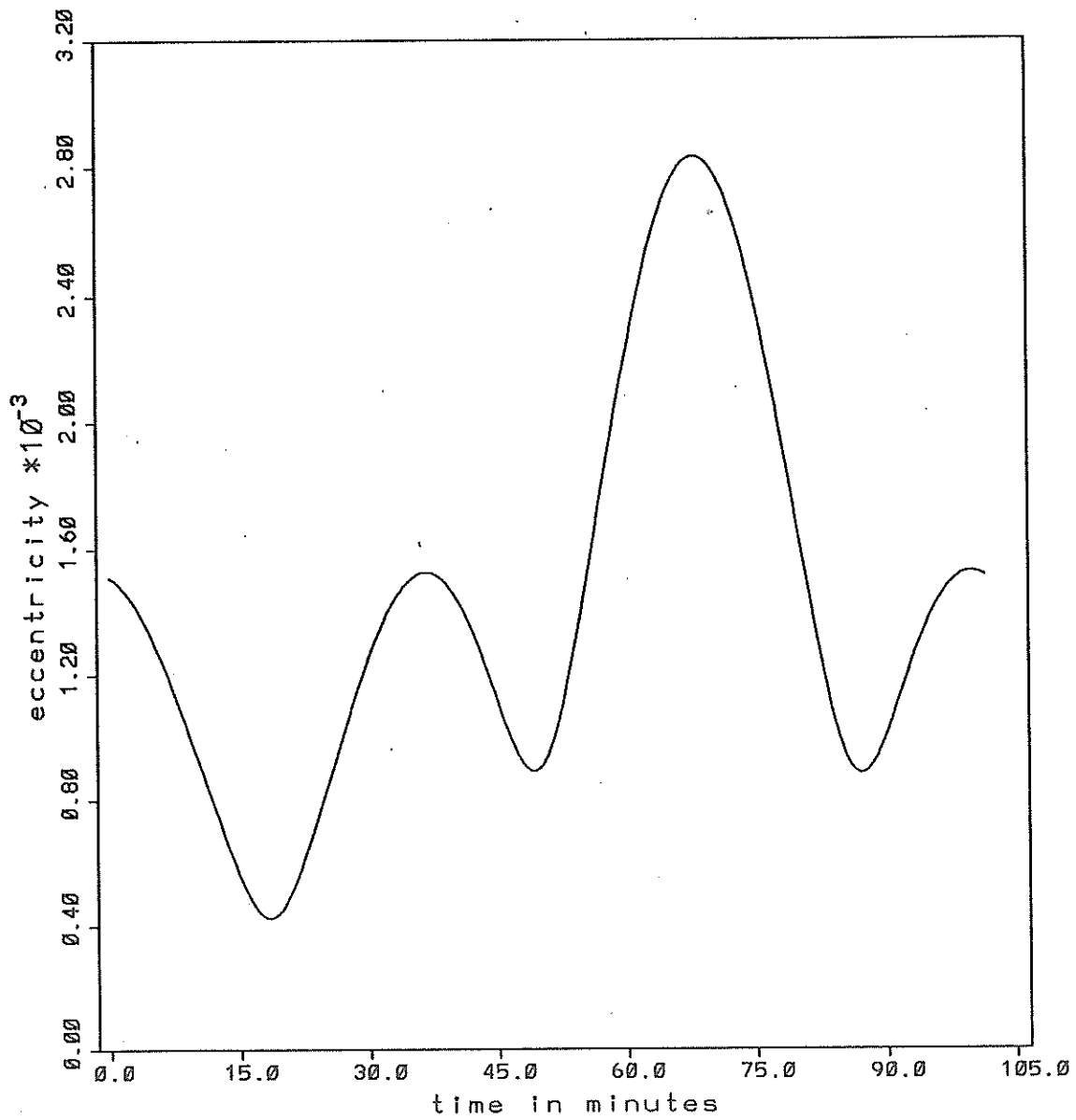
plot
  dimxy         0          6.0          8.0
  iorbpl        1
  iorb1         1 1 1
  iorb2         0 1 0
  iplt          2
  lintyp        0
  title1        orbit parameters for spot
  title2        June 23, June 28 1989
  end

finis
  
```

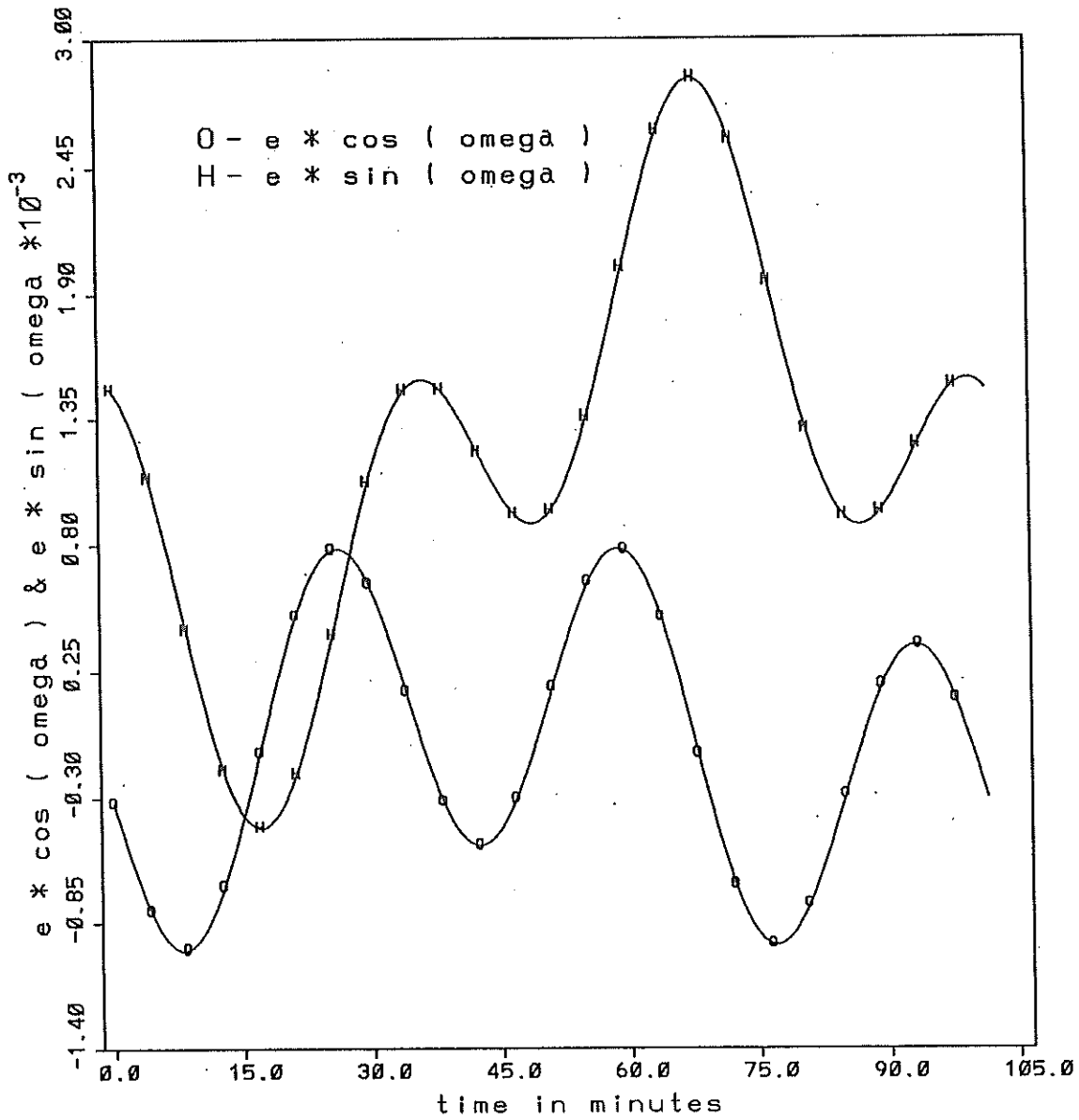
evolution of the semi-major axis along one orbit
spot June 23, 1989



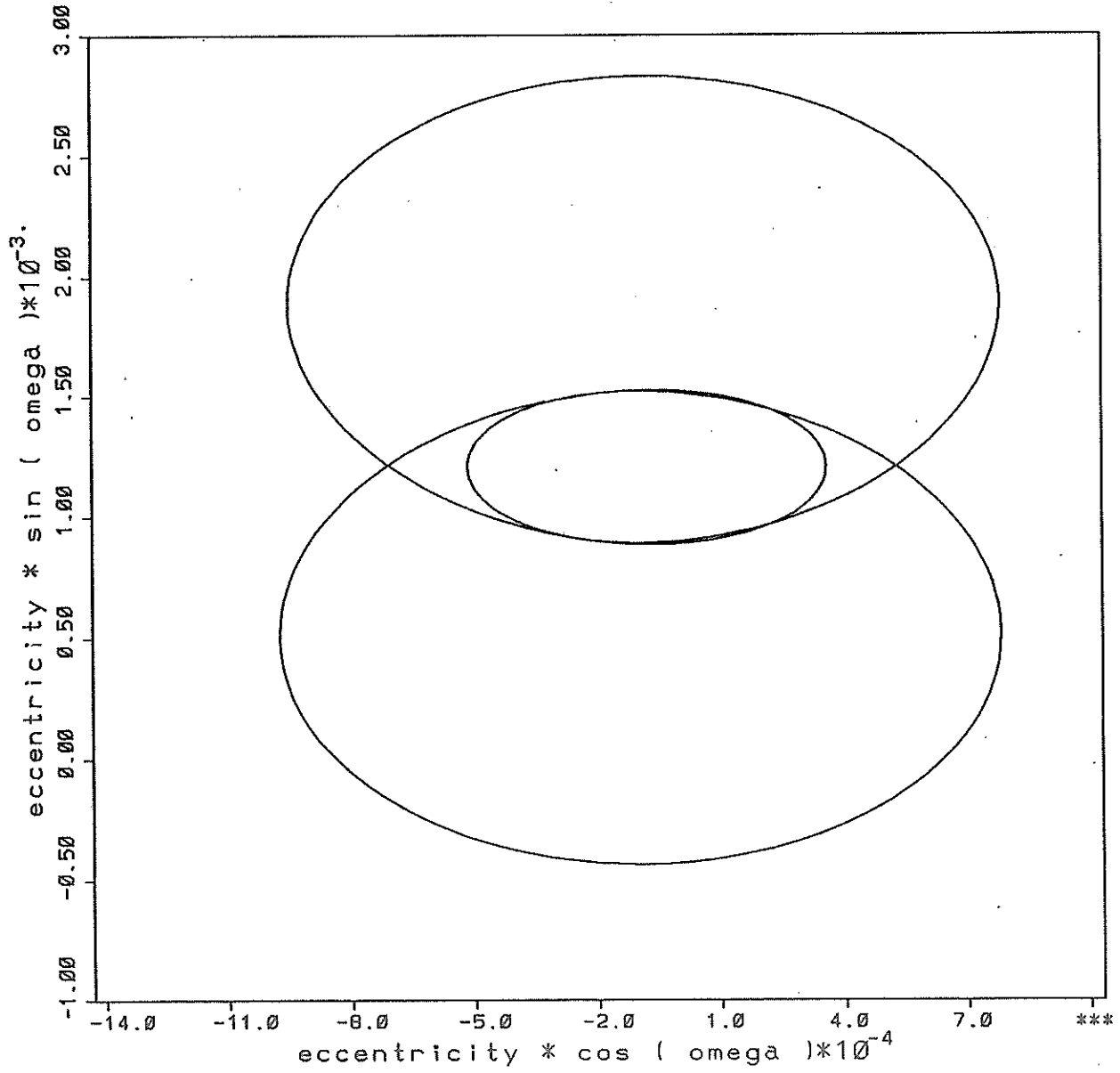
eccentricity along one orbit of spot
June 23, 1989



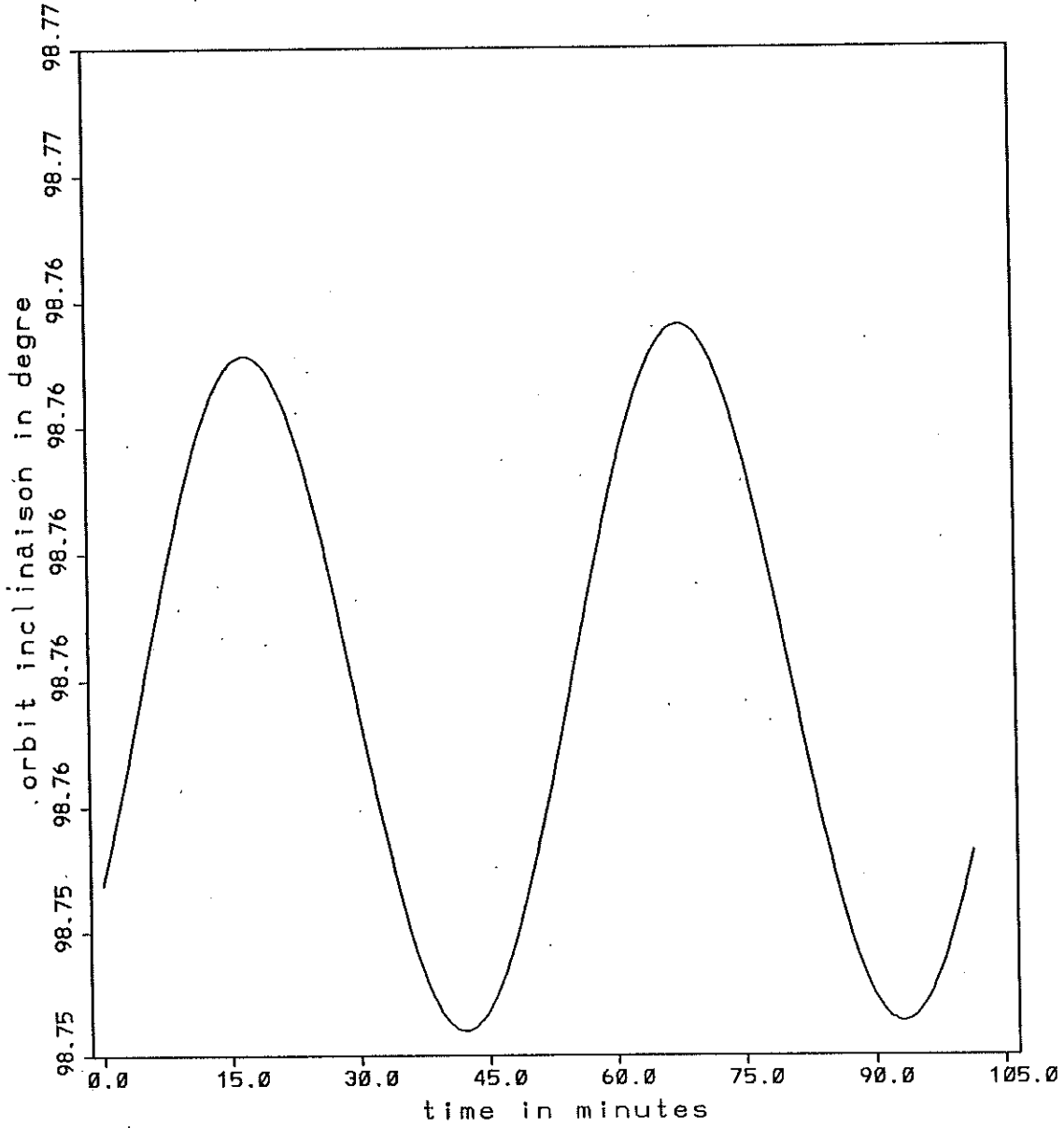
eccentricity vector of spot along one orbit
 June 23, 1989



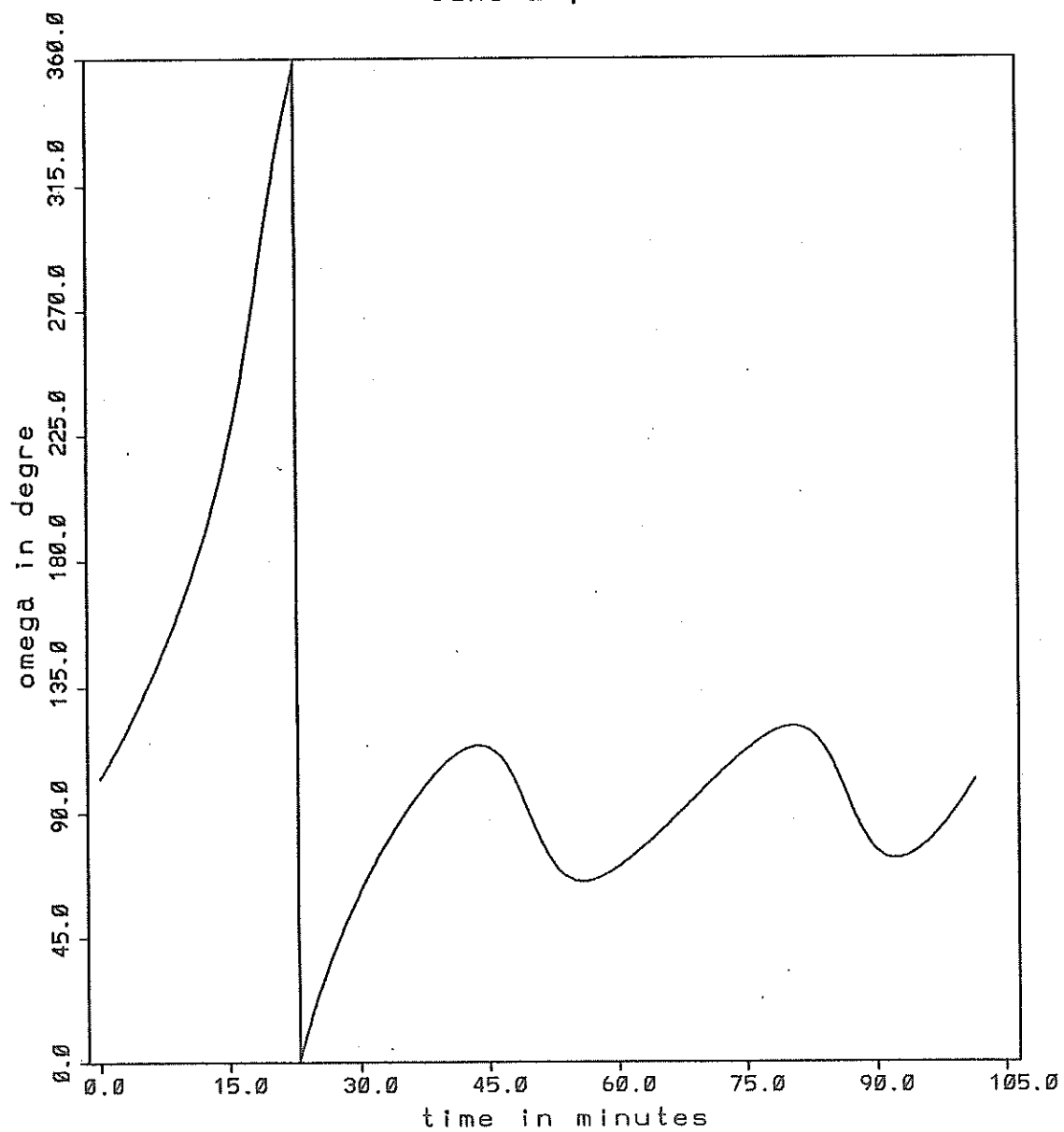
The 8 of the eccentricity vector
June 23, 1989



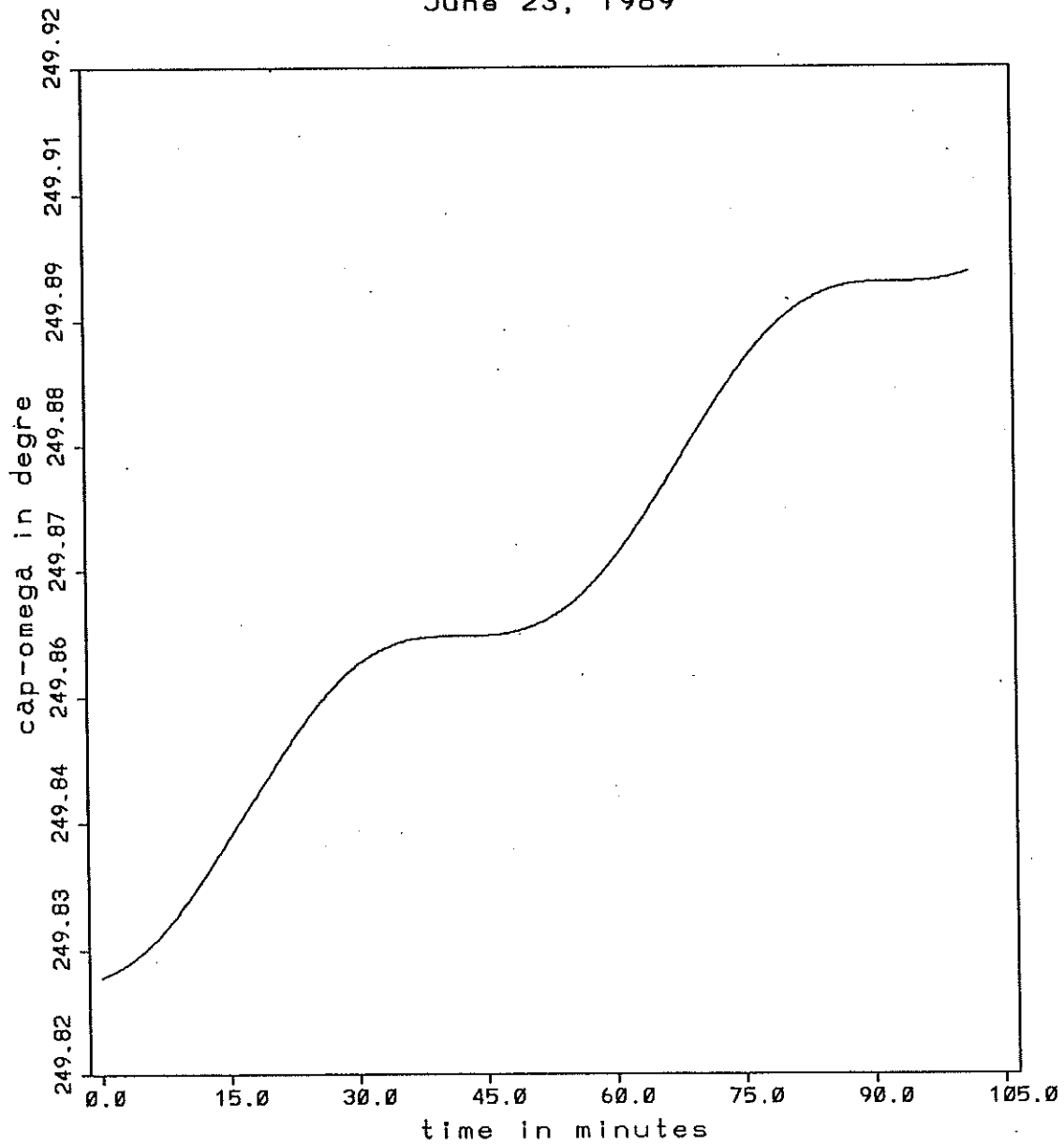
inclinaison along one orbit of spot
June 23, 1989



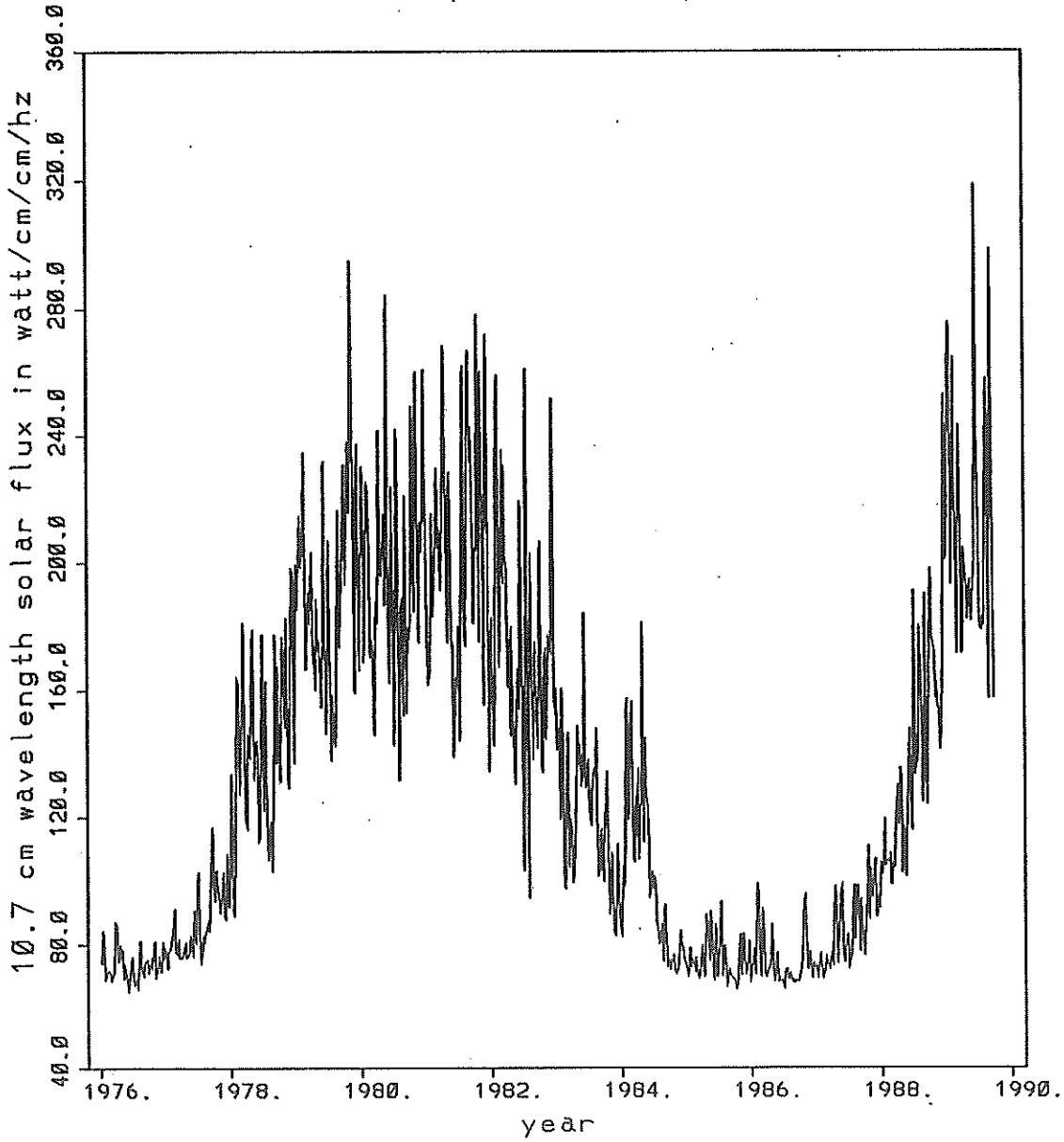
evolution of omega along one orbit of spot
June 23, 1989



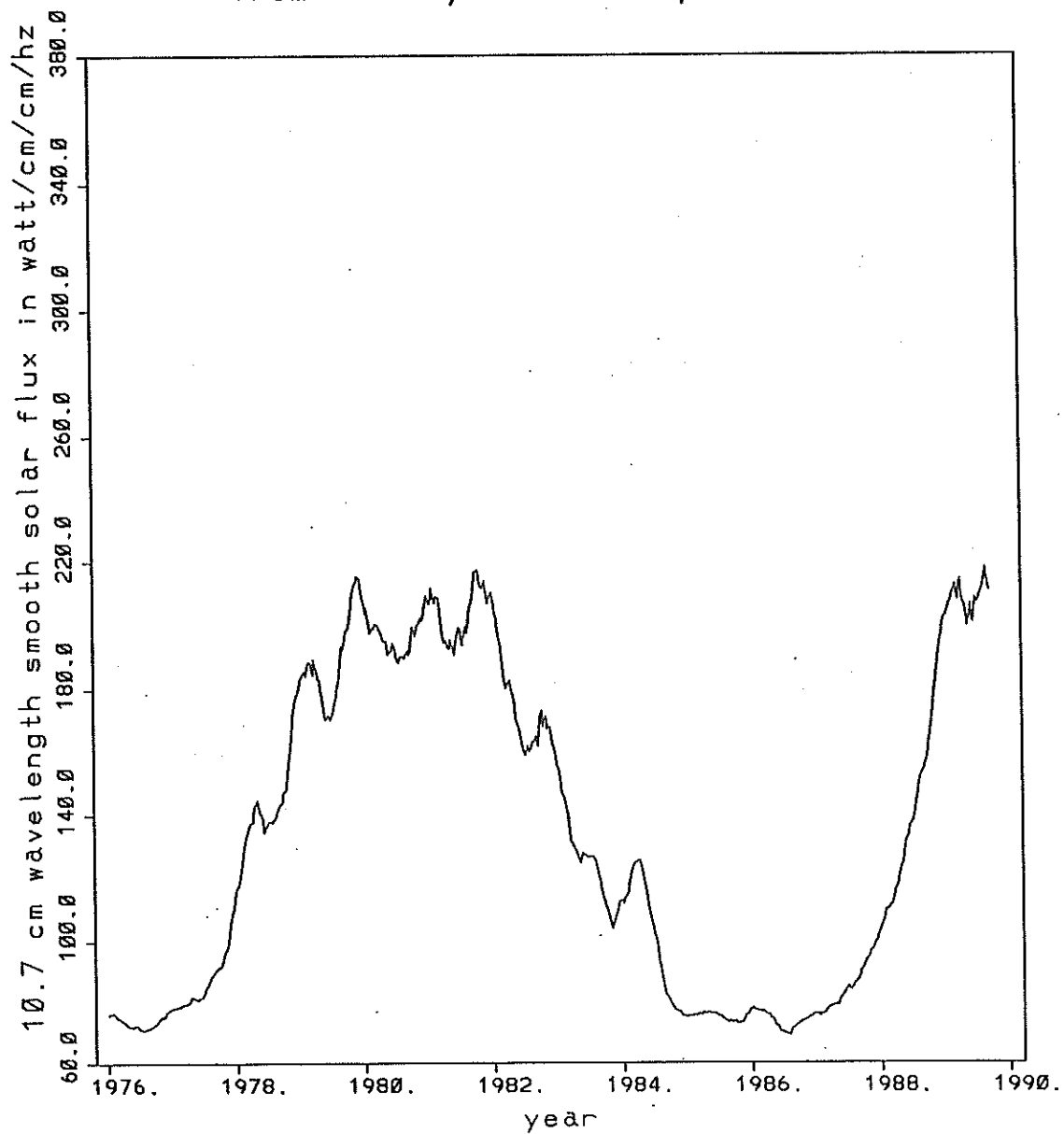
evolution of cap-omega along one orbit
June 23, 1989



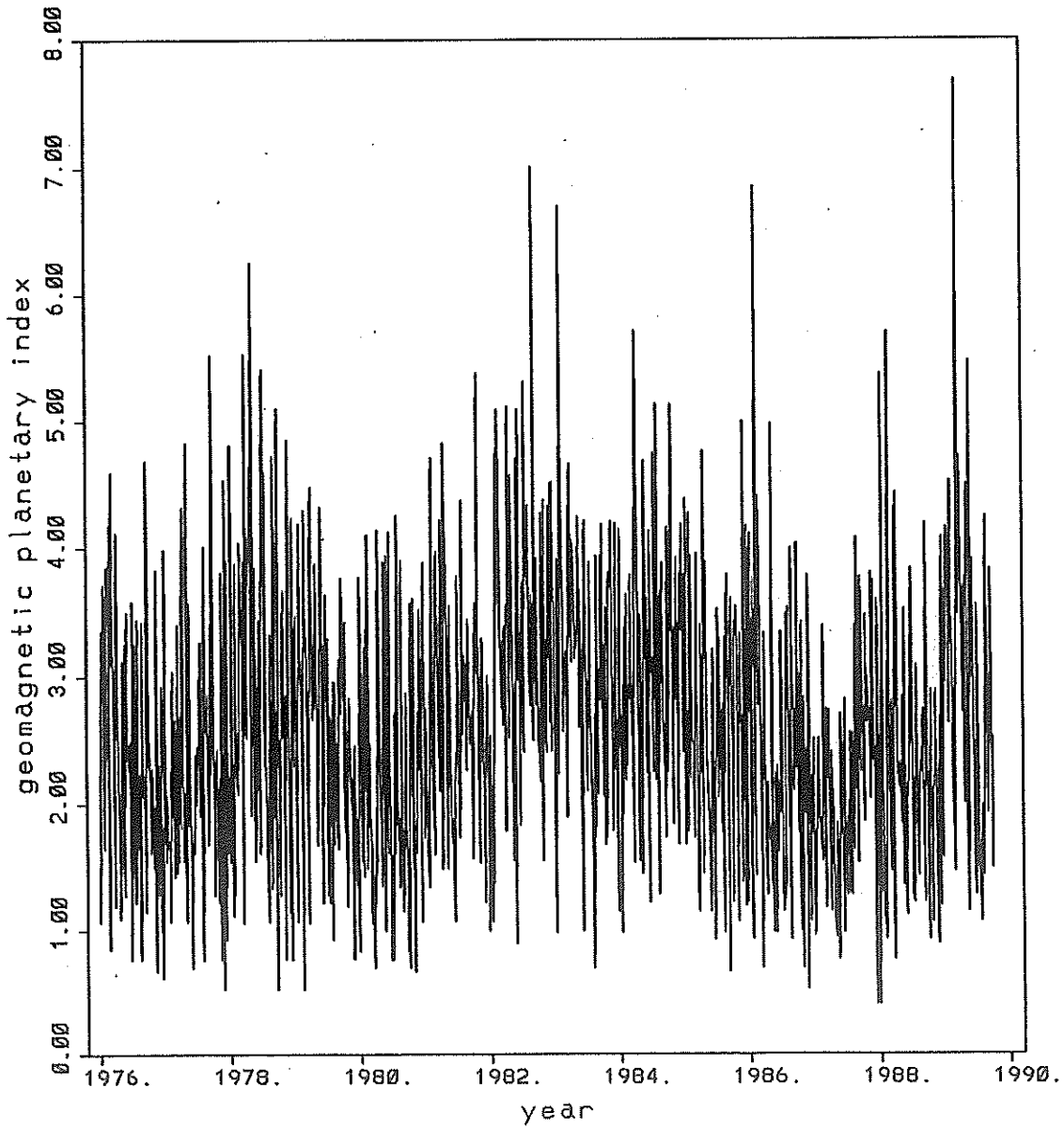
10.7 cm solar flux
from January 1976 to September 1989



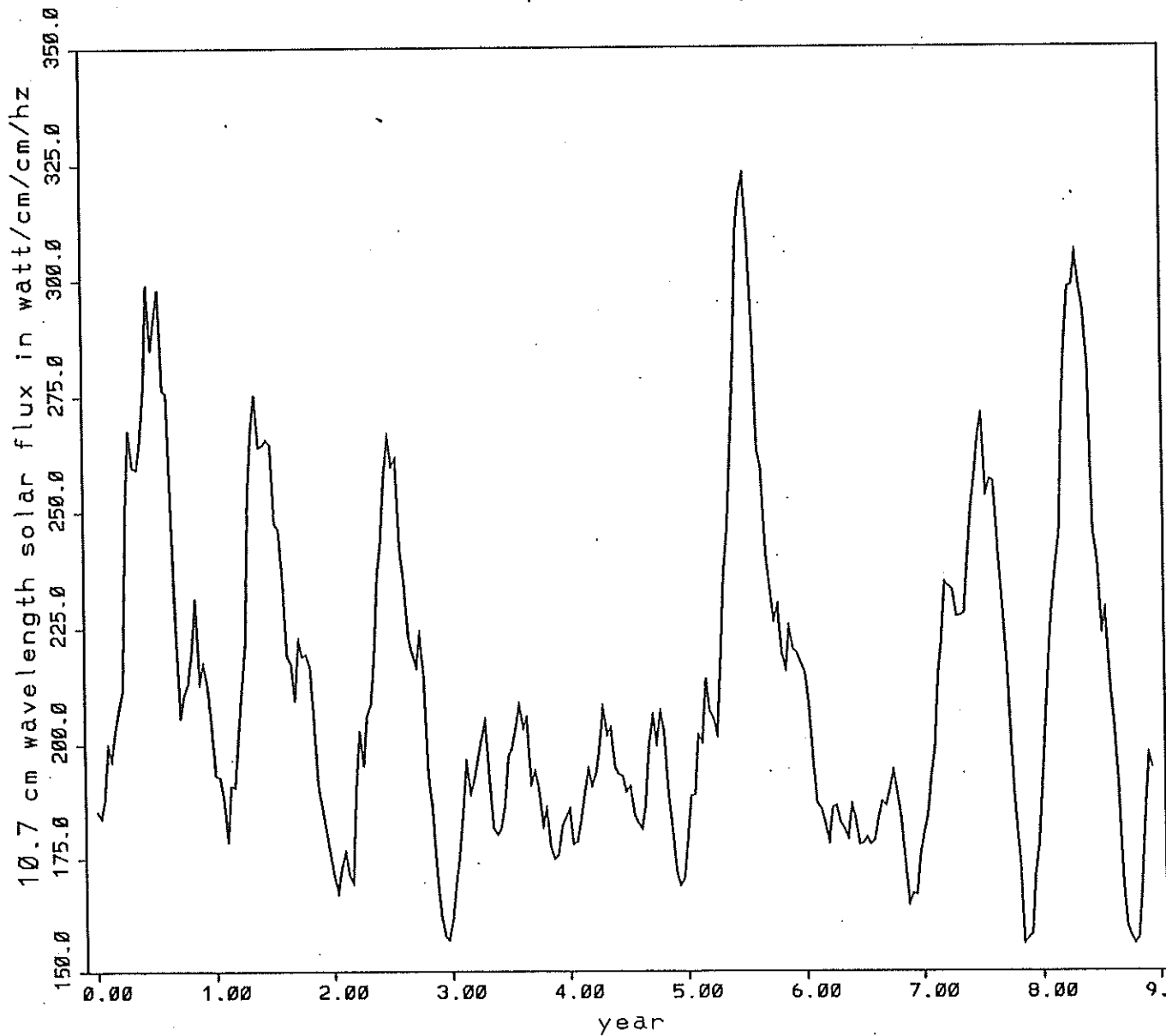
10.7 cm smooth solar flux
from January 1976 to September 1989



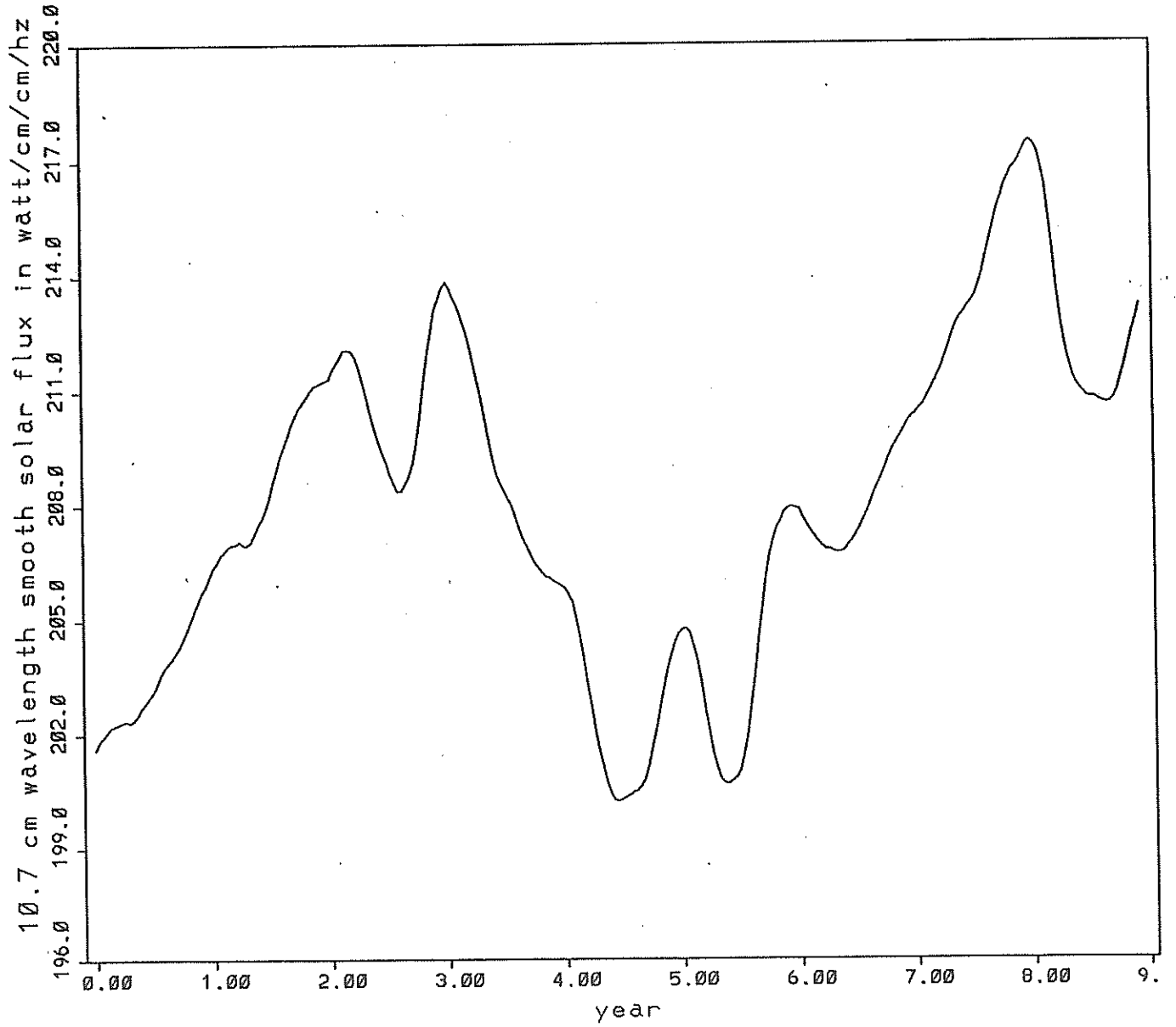
geomagnetic planetary index
from January 1976 to September 1989



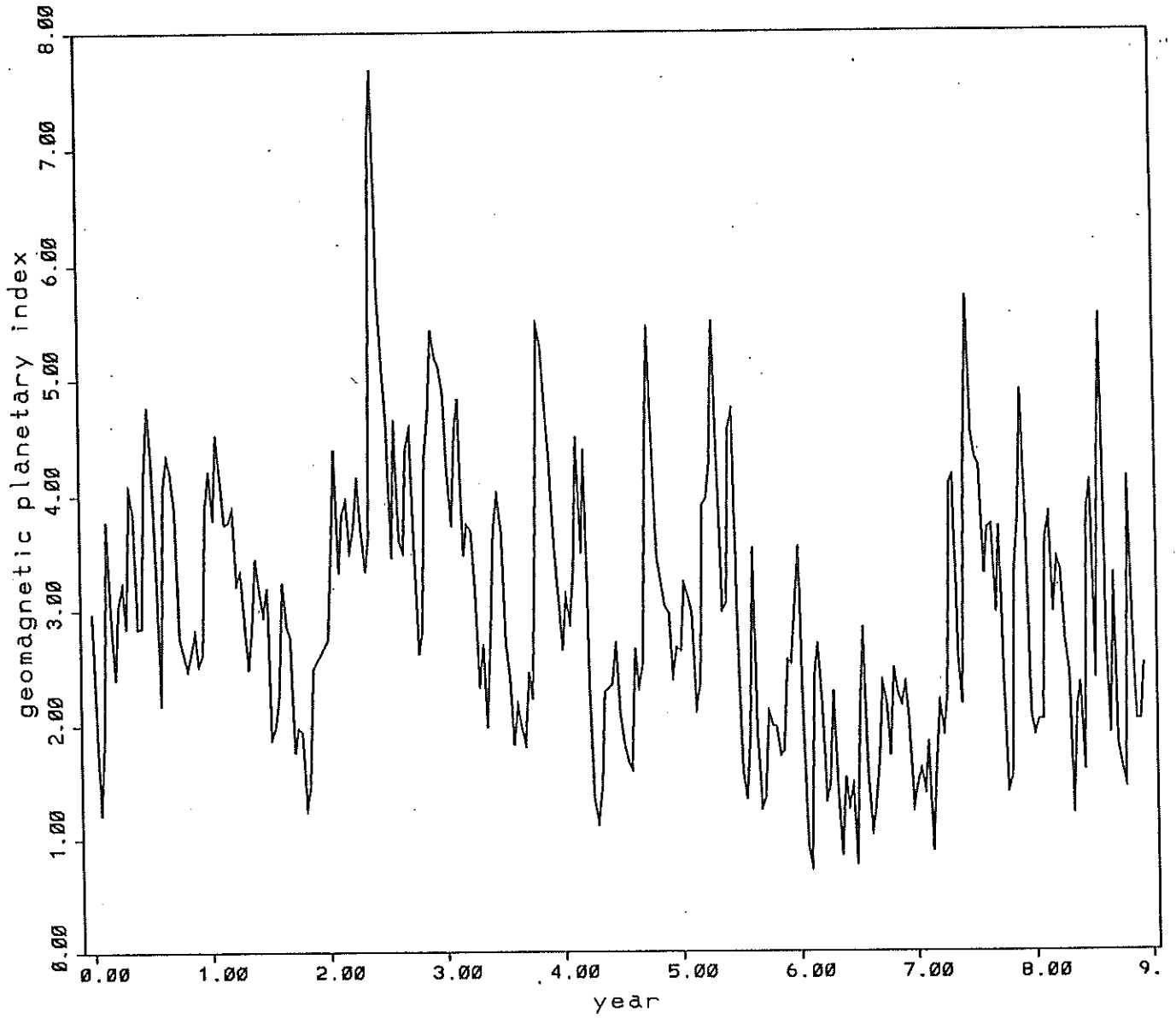
10.7 cm solar flux
from January 1989 to September 1989



10.7 cm smooth solar flux
from January 1989 to September 1989



geomagnetic planetary index
from January 1989 to September 1989



spot orbit simulation - un jour - atmospheric model : dtm -
 june 23,1989 - seulement drag -

W.1

```

initial
  epoch1          1989.0          6.0          23.0
  epoch2           5.             0.           24.
  pos      1 1      7205000.0      1.51d-3      98.7
  vel              102.50          249.70      287.0
  end

runmode
  mode      2
  iter      0
  end

forces
  satid          8600101.0          1850.0
  sun           0
  drag          5
  exfrot        3
  dtides        0
  etper         0
  polmot        0
  relprt        0
  spdut1        0
  geo           1 0
  gm            398600.436
  moon          0
  mmax          0
  njmax         0
  nmax          0
  extrad        0
  shadow        0          6378137.0
  end

integ/out
  tstart         0.
  tfdays        1.
  nprint 100
  dtnew         179.0
  end

sta/obs
  chordp
  measx      1
  end

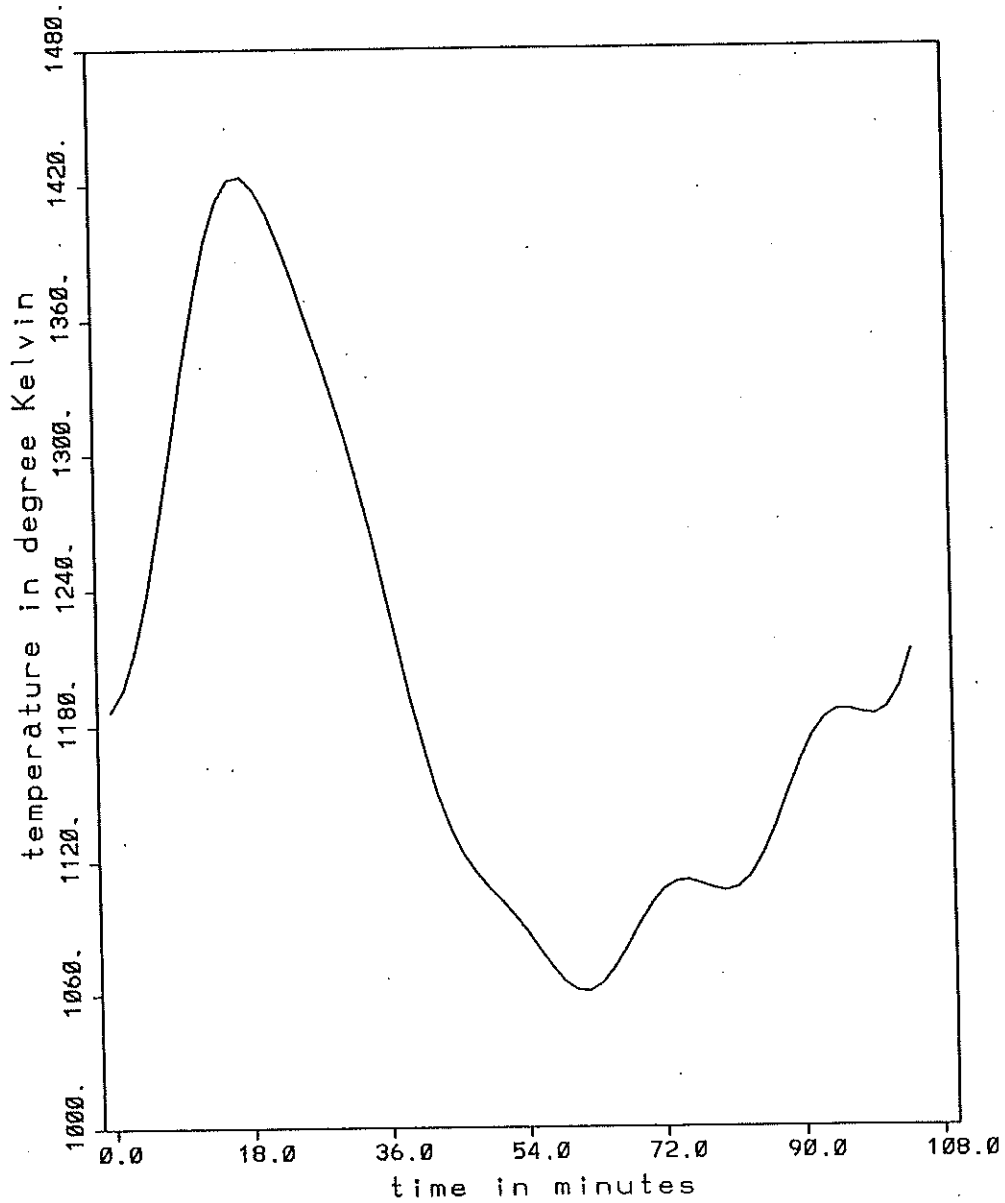
files
  densit      1
  accsat     0 1
  pltdat     1
  end

plot
  dimxy      0          6.0          8.0
  iorbpl     1
  iorb1      1 1 1
  iorb2      0 1 0
  iplt       2
  lintyp     0
  title1     orbit parameters for spot
  title2     June 23, June 28 1989
  end

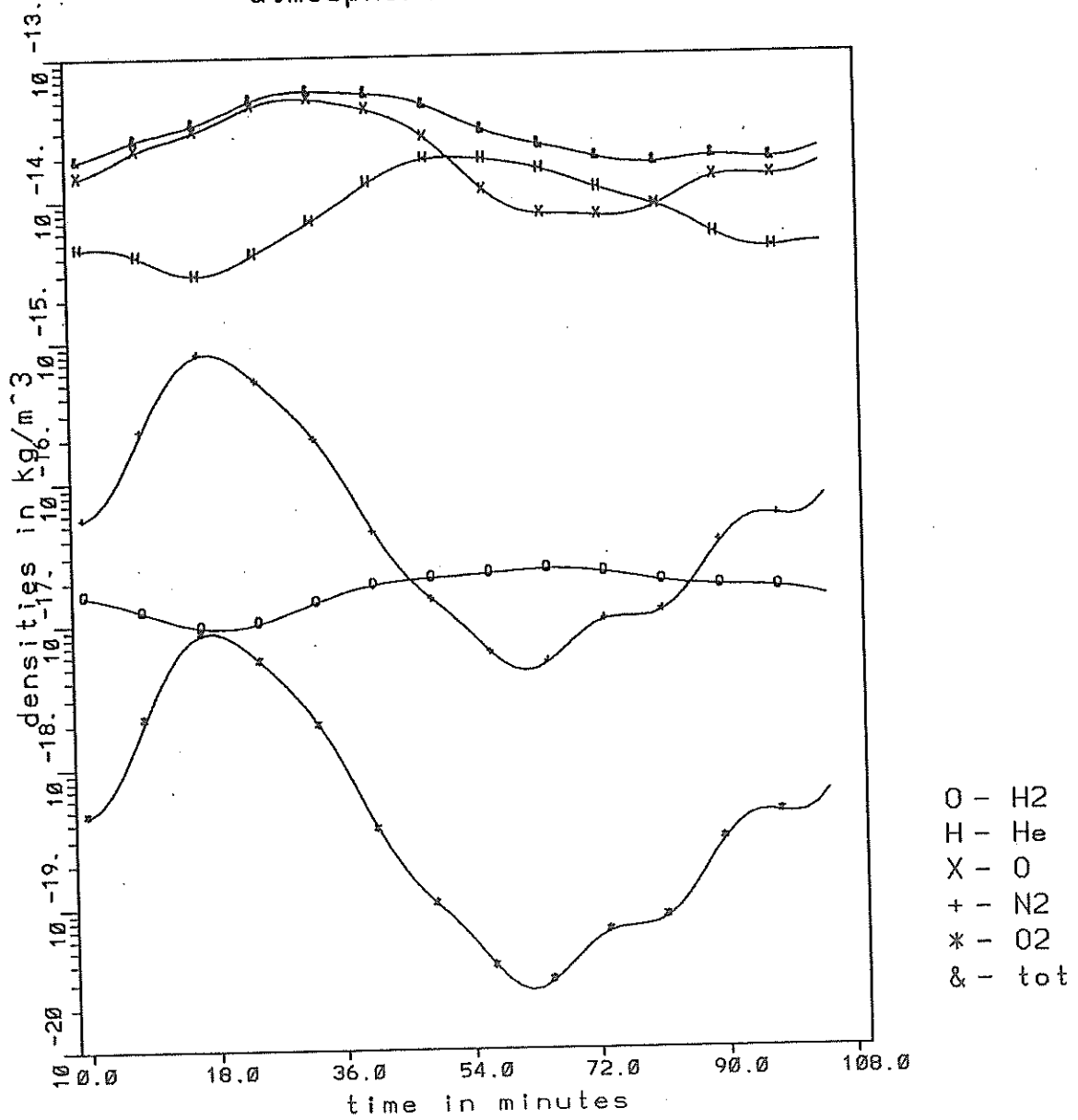
finis

```

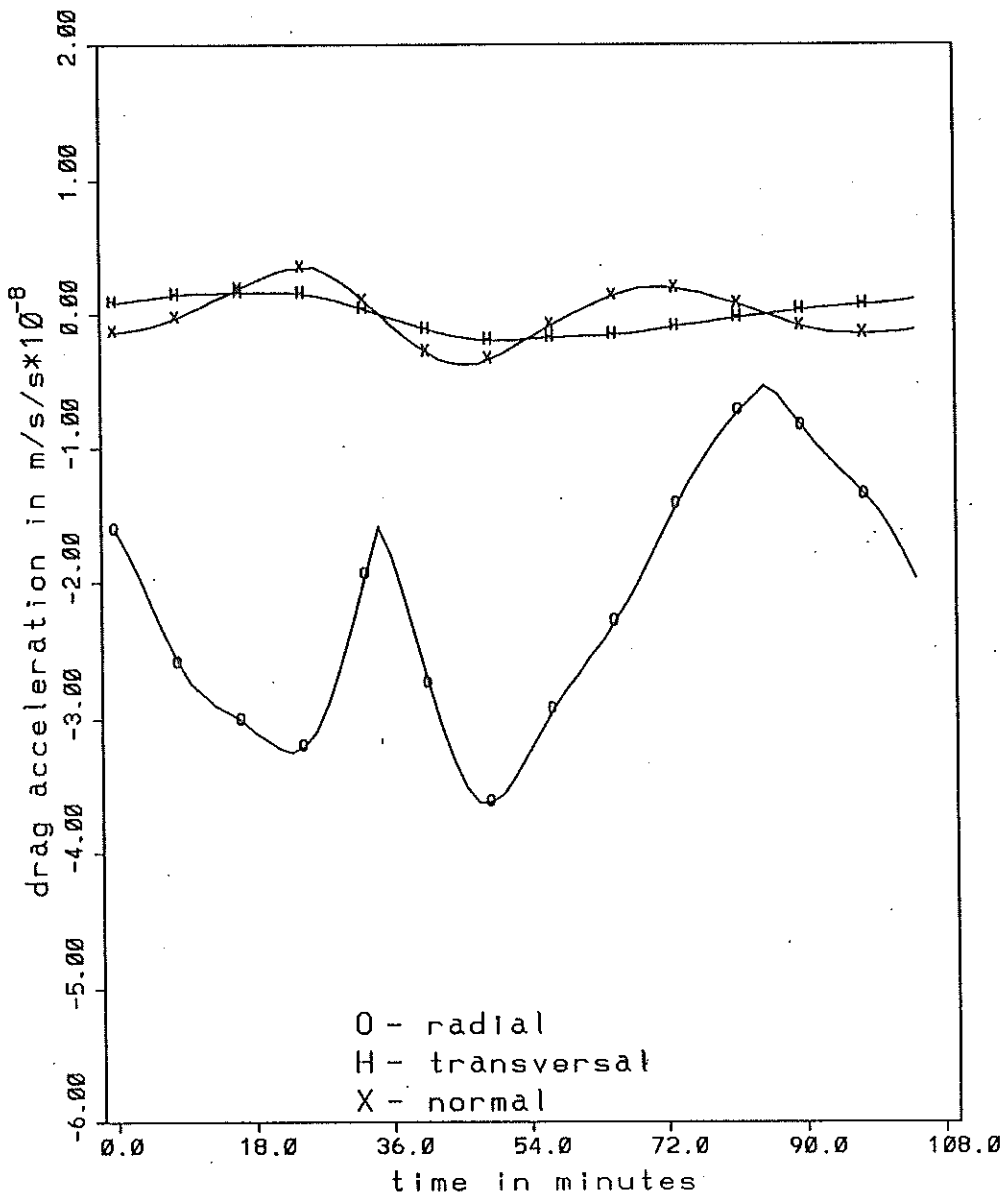
atmospheric temperature along one orbit of spot June 23, 1989
atmospheric model : dtm



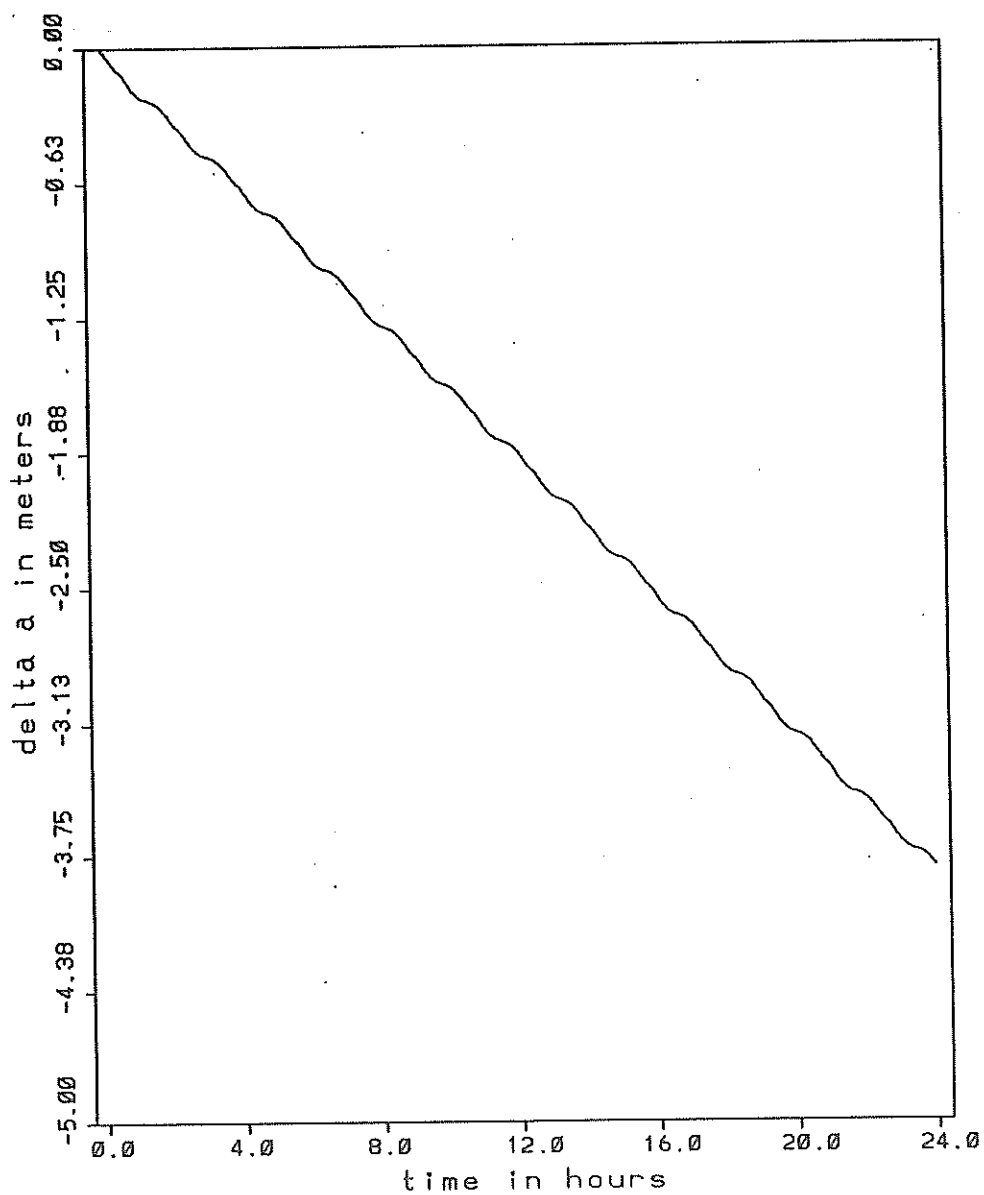
atmospheric densities along one orbit of spot June 23, 1989
 atmospheric model . dtm



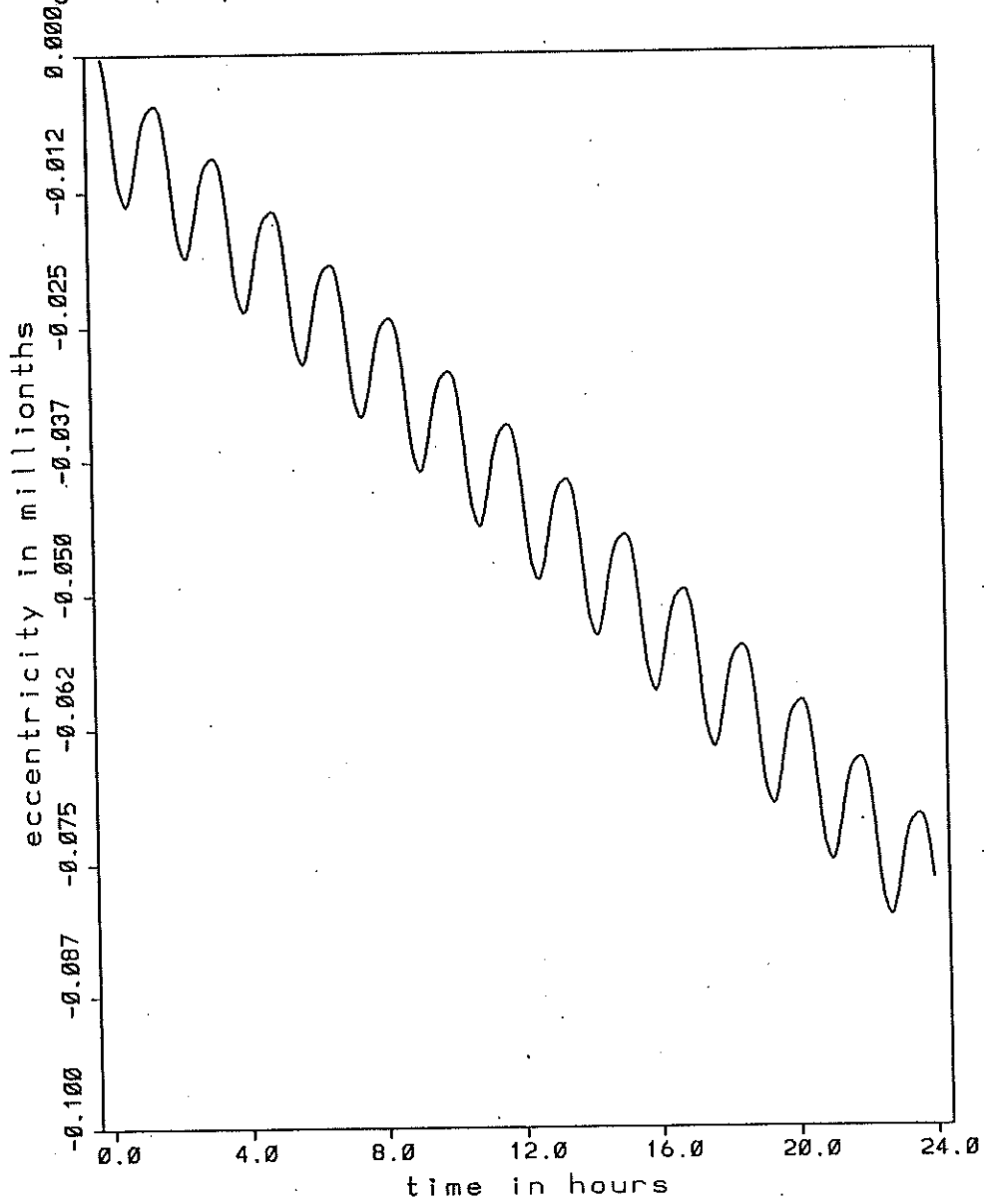
drag acceleration along one orbit of spot June 23, 1989
atmospheric model : dtm



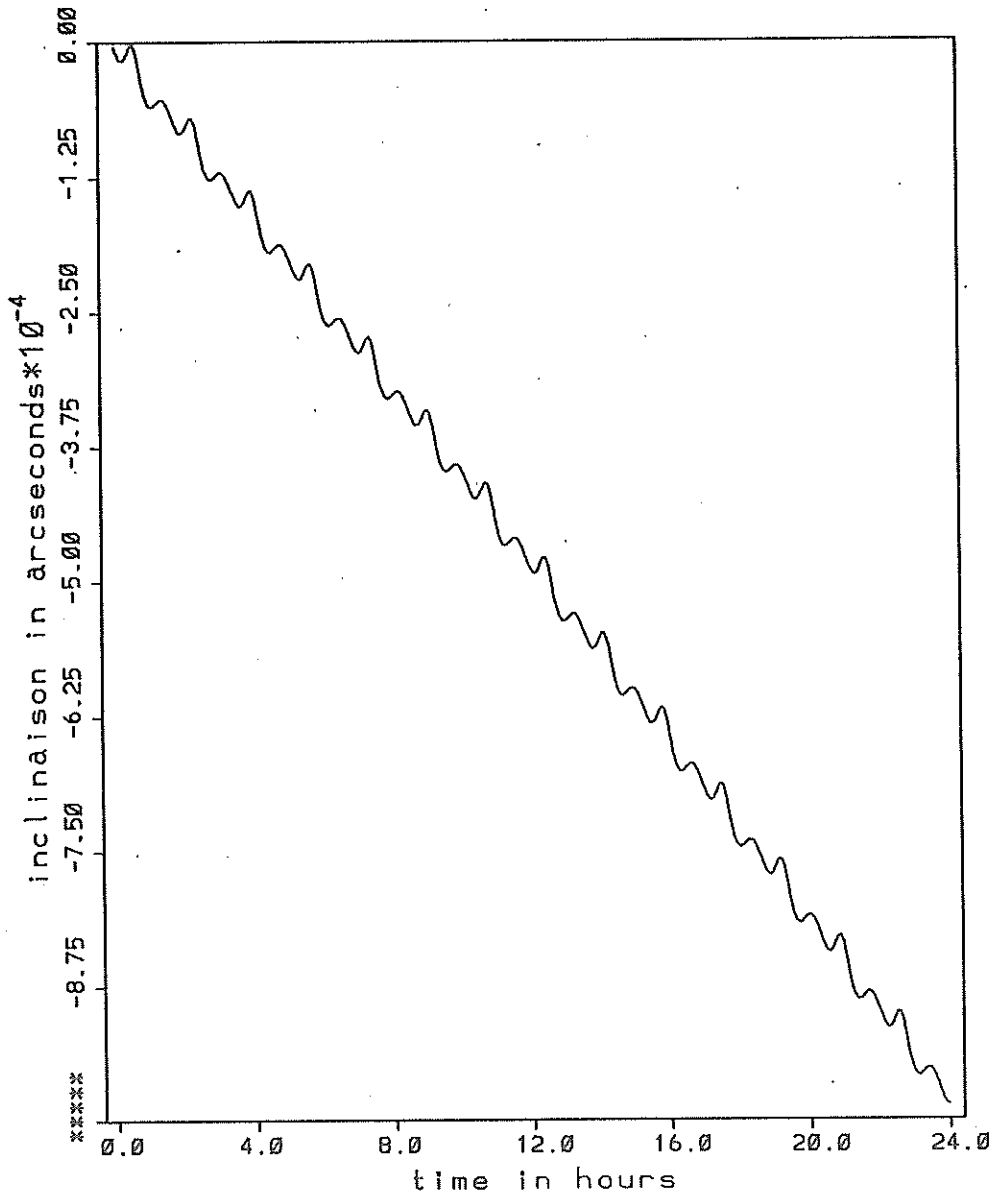
atmospheric drag influence on the semi-major-axis - dtm -
during one day-arc of spot on June 23, 1989 - only drag -



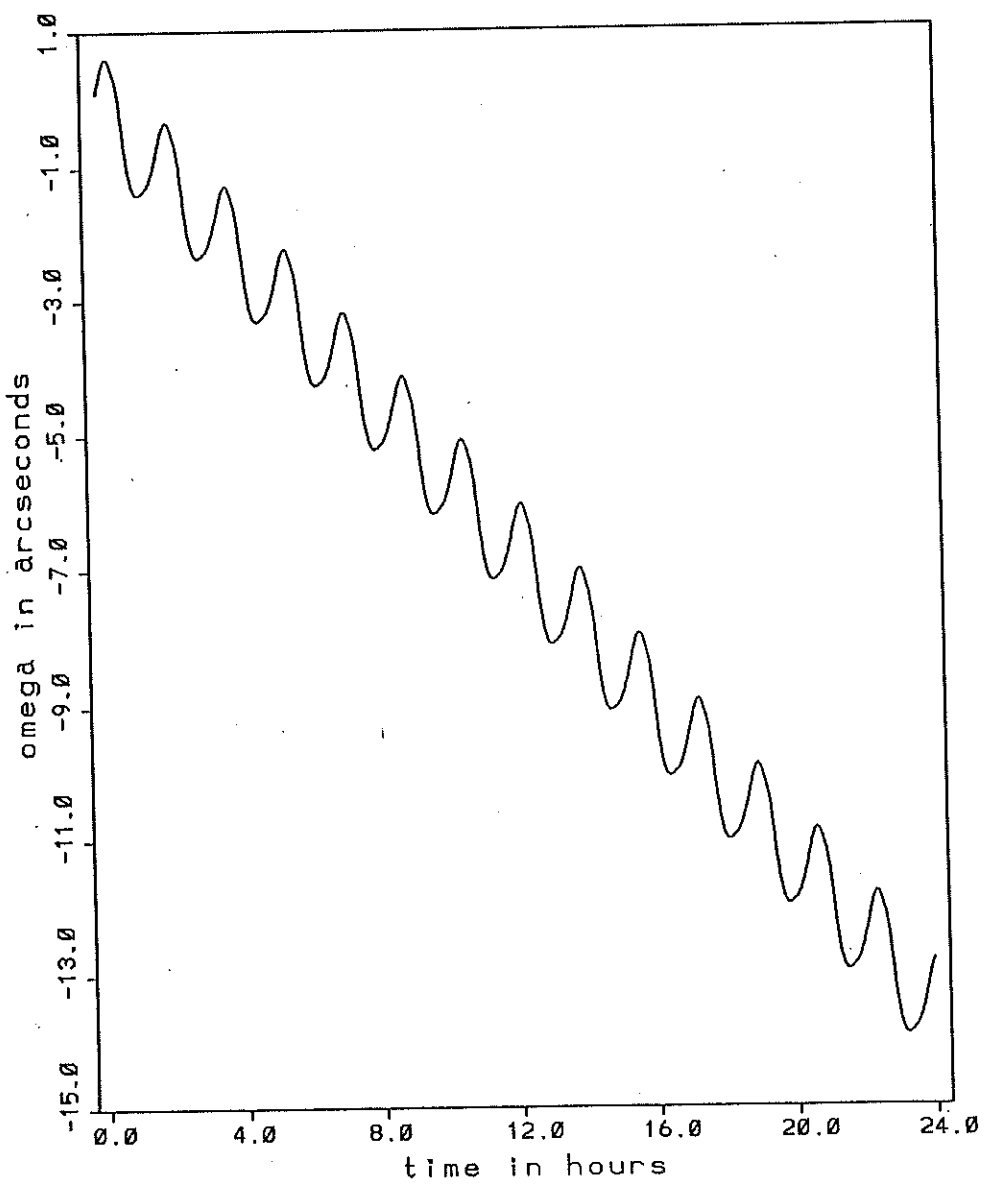
atmospheric drag influence on the eccentricity - dtm -
during one day-arc of spot on June 23, 1989 - only drag -



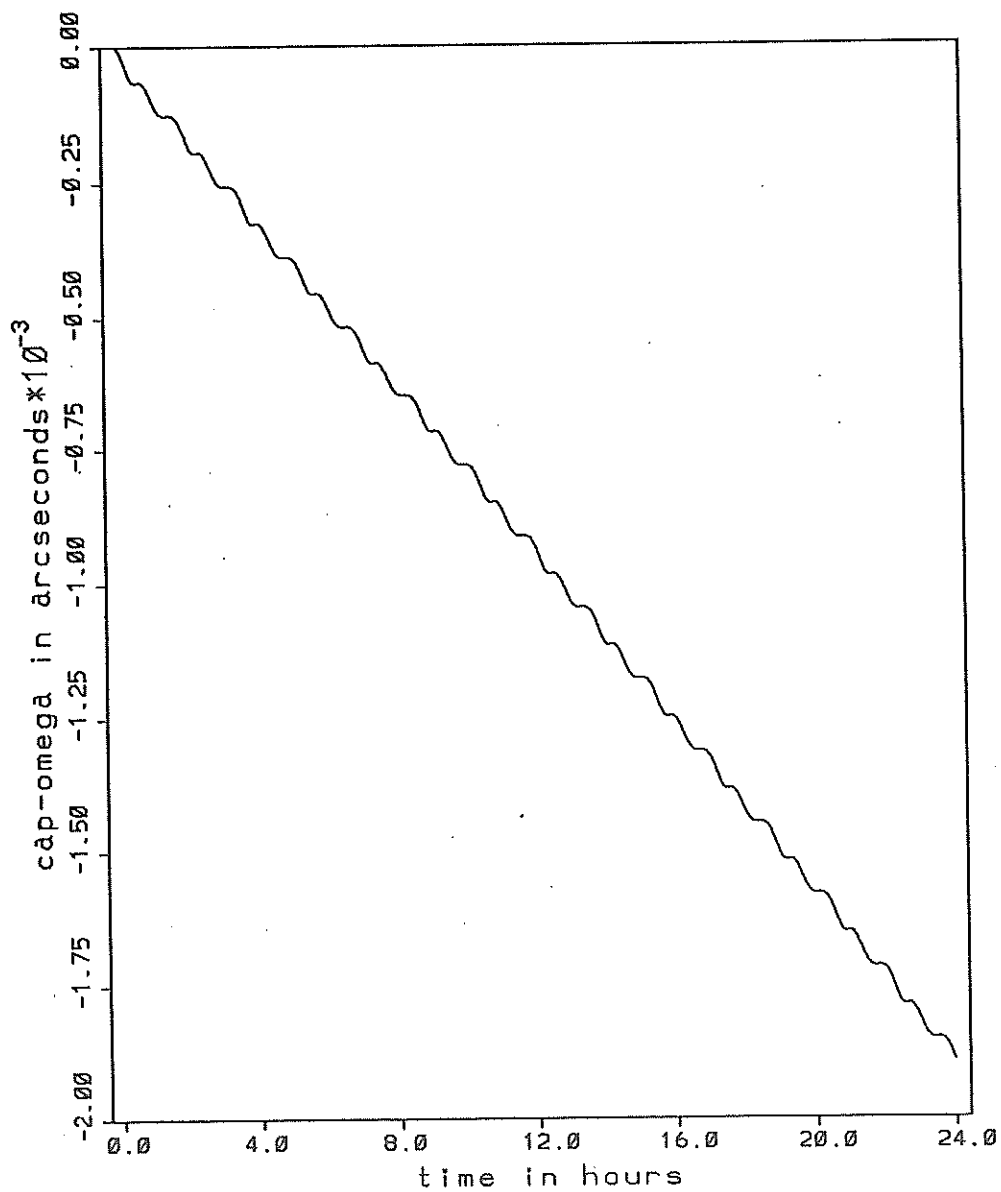
atmospheric drag influence on inclinasion - dtm -
during one day-arc of spot on June 23, 1989 - only drag -



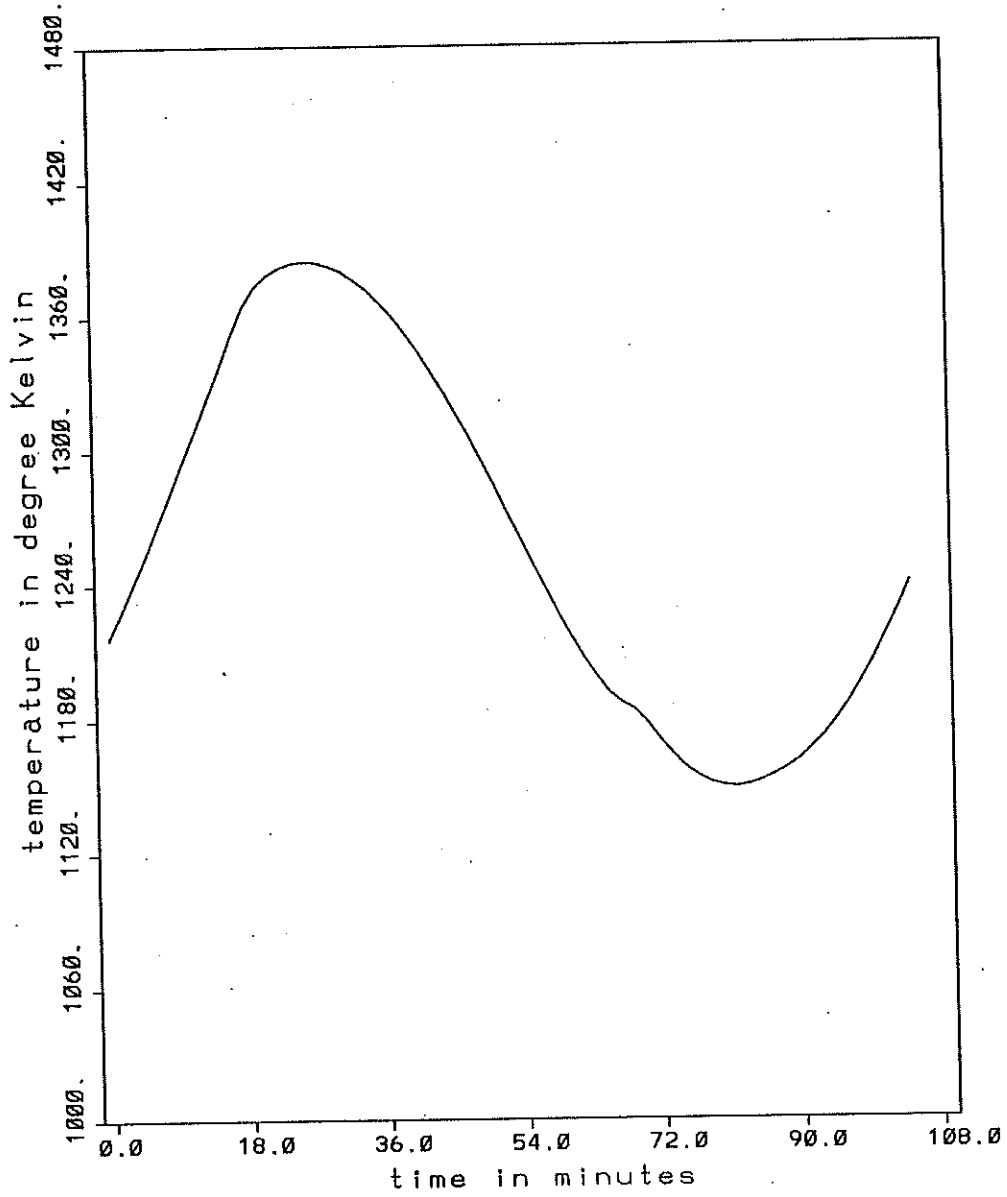
atmospheric drag influence on omega - dtm -
during one day-arc of spot on June 23, 1989 - drag only -



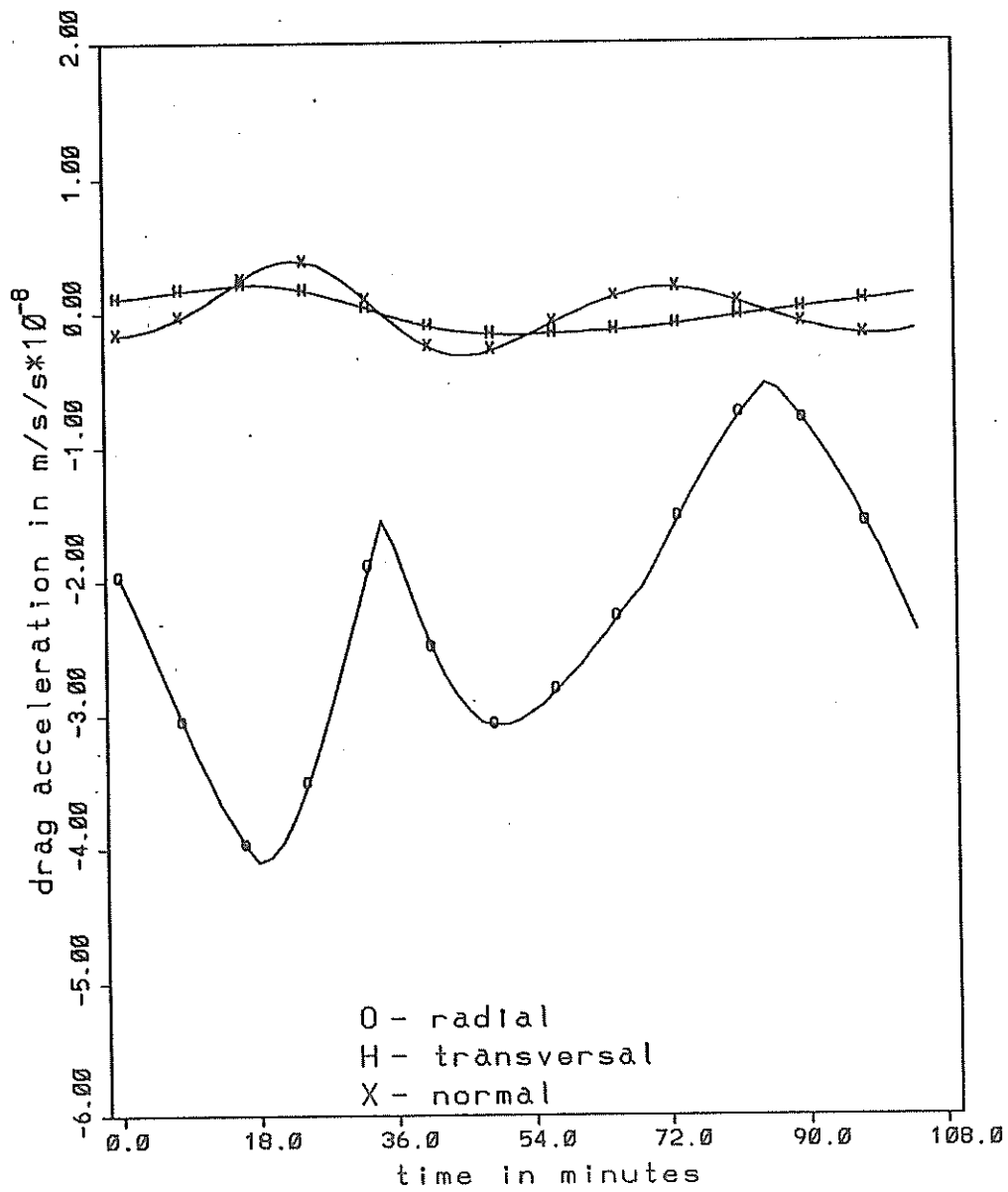
atmospheric drag influence on cap-omega - dtm -
during one day-arc of spot on June 23, 1989 - only drag -



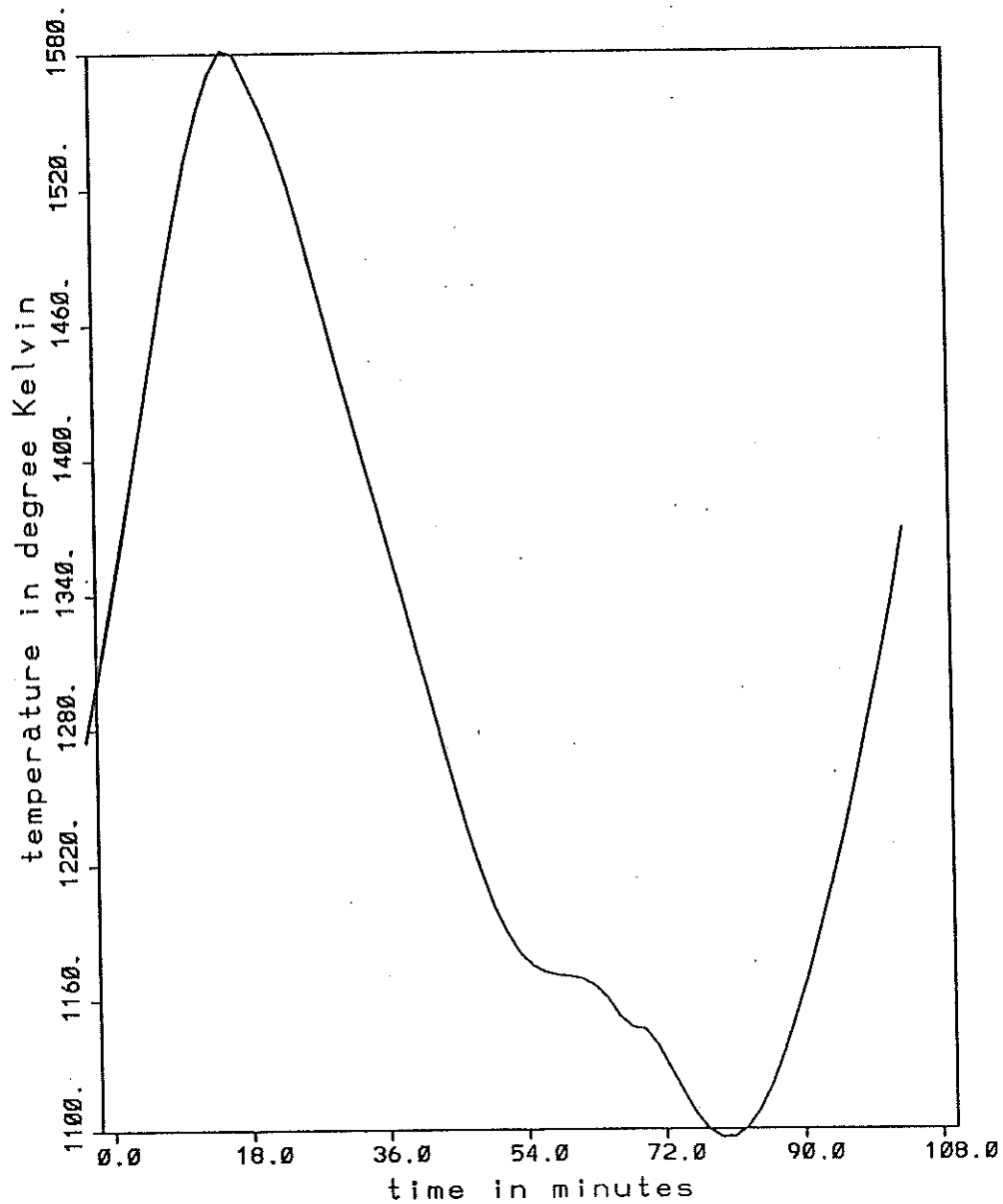
atmospheric temperature along one orbit of spot June 23, 1989
atmospheric model : jacchia 1971



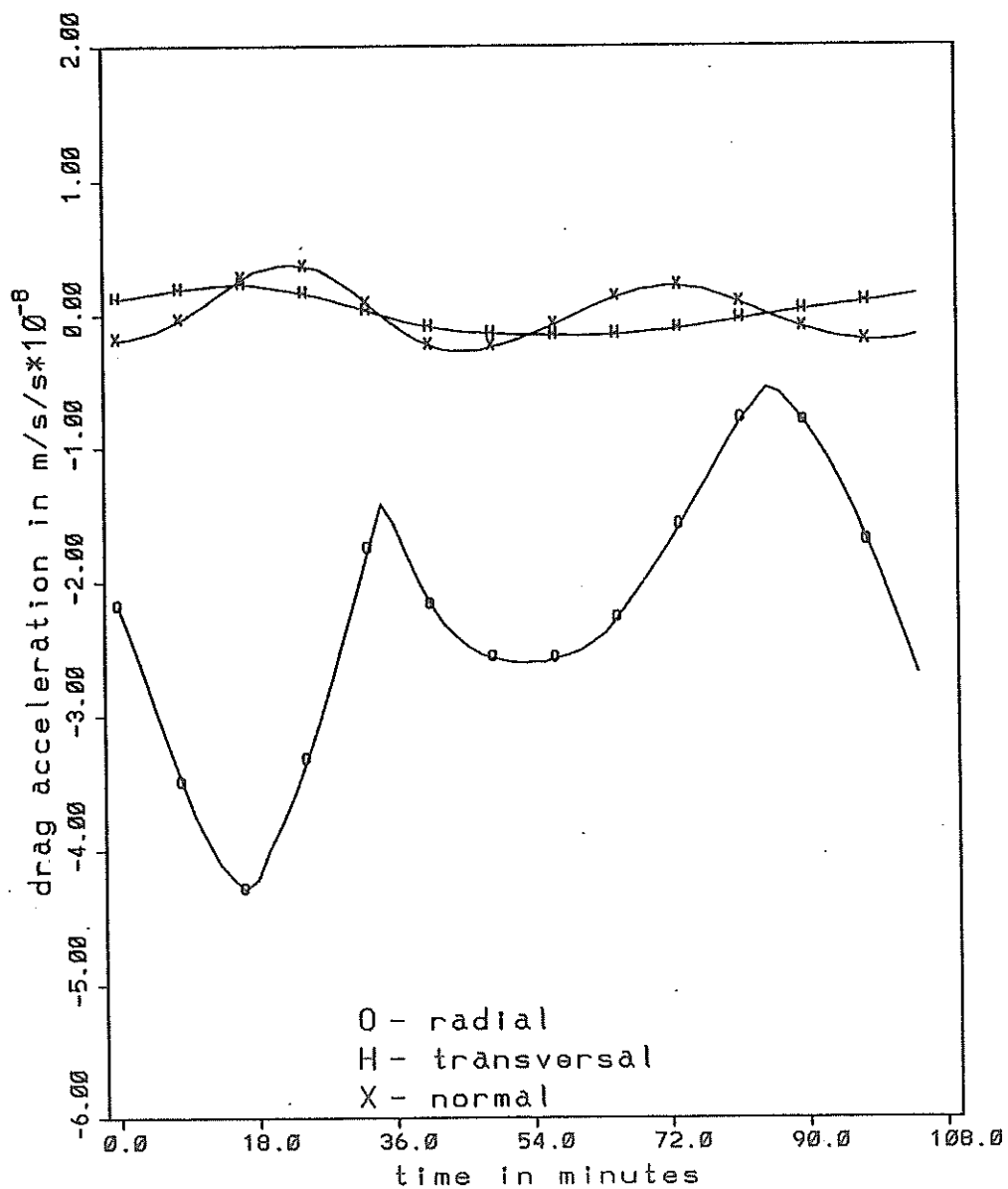
drag acceleration along one orbit of spot June 23, 1989
atmospheric model : jacchia 1971



atmospheric temperature along one orbit of spot June 23, 1989
atmospheric model : jacchia 1977



drag acceleration along one orbit of spot June 23, 1989
atmospheric model : jacchia 1977



date	Atm. Model	ssof	solar flux	Kp	Δa (m)	ΔM (min)
June 23, 1989	Jacchia 71	206.0	233.7	1.3	3.99	0.272
June 23, 1989	Jacchia 77	206.0	233.7	1.3	3.77	0.195
June 23, 1989	dtm	206.0	233.7	1.3	3.80	0.350
June 17, 1989	dtm	201.4	322.5	1.6	3.66	0.275
March 14, 1989	dtm	209.7	243.3	7.7	9.85	0.948
Nov. 22, 1986	dtm	74.9	74.4	0.5	0.26	0.028
Sept. 1, 1989	dtm	217.5	190.6	1.9	3.11	0.291
May 9, 1989	dtm	202.0	199.0	1.3	2.68	0.246
Oct. 10, 1985	dtm	72.9	65.9	1.6	0.27	0.285
Dec. 28, 1987	dtm	100.8	103.3	0.35	0.39	0.040
Feb. 22, 1988	dtm	108.8	110.3	5.7	0.87	0.065
Feb. 26, 1988	dtm	101.3	110.5	2.4	0.52	0.049

Variation of the Semi-Major Axis Δa in Meter
for one Day Orbit of SPOT
with Atmospheric Model 'Jacchia 71'

				geomagnetic index Kp		
				Δa (m)	1.0	4.0
sof	50	ssof	100	0.25	0.37	0.65
	100		150	0.60	0.96	1.84
	150		200	1.70	2.73	4.99

				geomagnetic index Kp		
				Δa (m)	1.0	4.0
ssof = sof			100	0.35	0.53	0.98
			150	0.90	1.46	2.79
			200	2.58	4.06	7.13

				geomagnetic index Kp		
				Δa (m)	1.0	4.0
sof	150	ssof	100	0.50	0.78	1.49
	200		150	1.37	2.23	4.14
	250		200	3.84	5.89	9.92

Variation of the Semi-Major Axis Δa in Meter
for one Day Orbit of SPOT
with Atmospheric Model 'Jacchia 77'

				geomagnetic index Kp		
				Δa (m)	1.0	4.0
sof	50	ssof	100	0.22	0.25	0.42
	100		150	0.31	0.77	1.28
	150		200	1.82	2.02	3.08

				geomagnetic index Kp			
				Δa (m)	1.0	4.0	7.0
ssof = sof				100	0.46	0.51	0.87
				150	1.22	1.36	2.16
				200	2.87	3.15	4.59

				geomagnetic index Kp		
				Δa (m)	1.0	4.0
sof	150	ssof	100	0.79	0.89	1.47
	200		150	1.95	2.16	3.27
	250		200	4.18	4.51	6.35

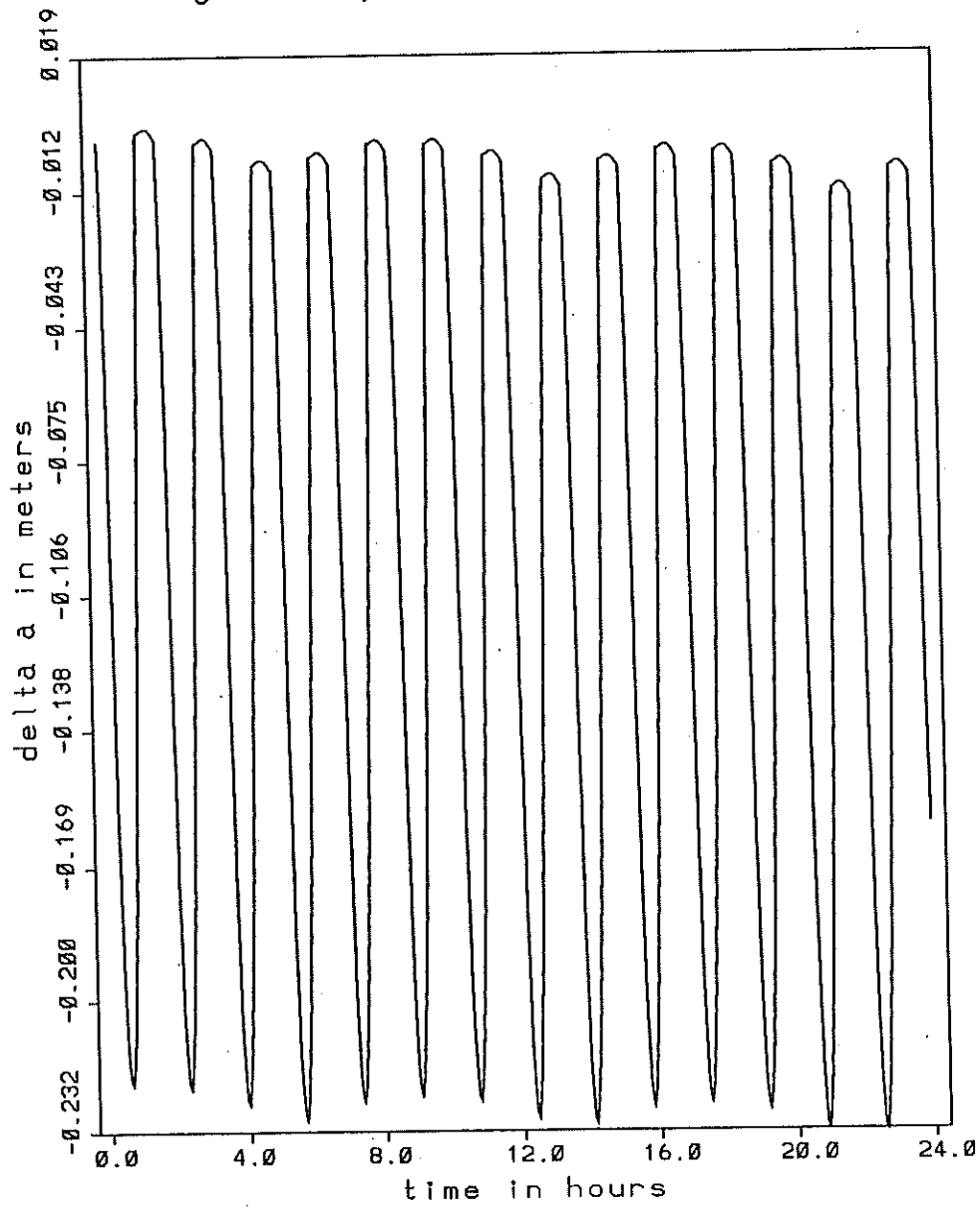
Variation of the Semi-Major Axis Δa in Meter
for one Day Orbit of SPOT
with Atmospheric Model 'dtm'

				geomagnetic index Kp		
				Δa (m)	1.0	4.0
sof	50	ssof	100	0.25	0.34	0.51
	100		150	0.56	0.83	1.30
	150		200	1.43	2.23	3.53

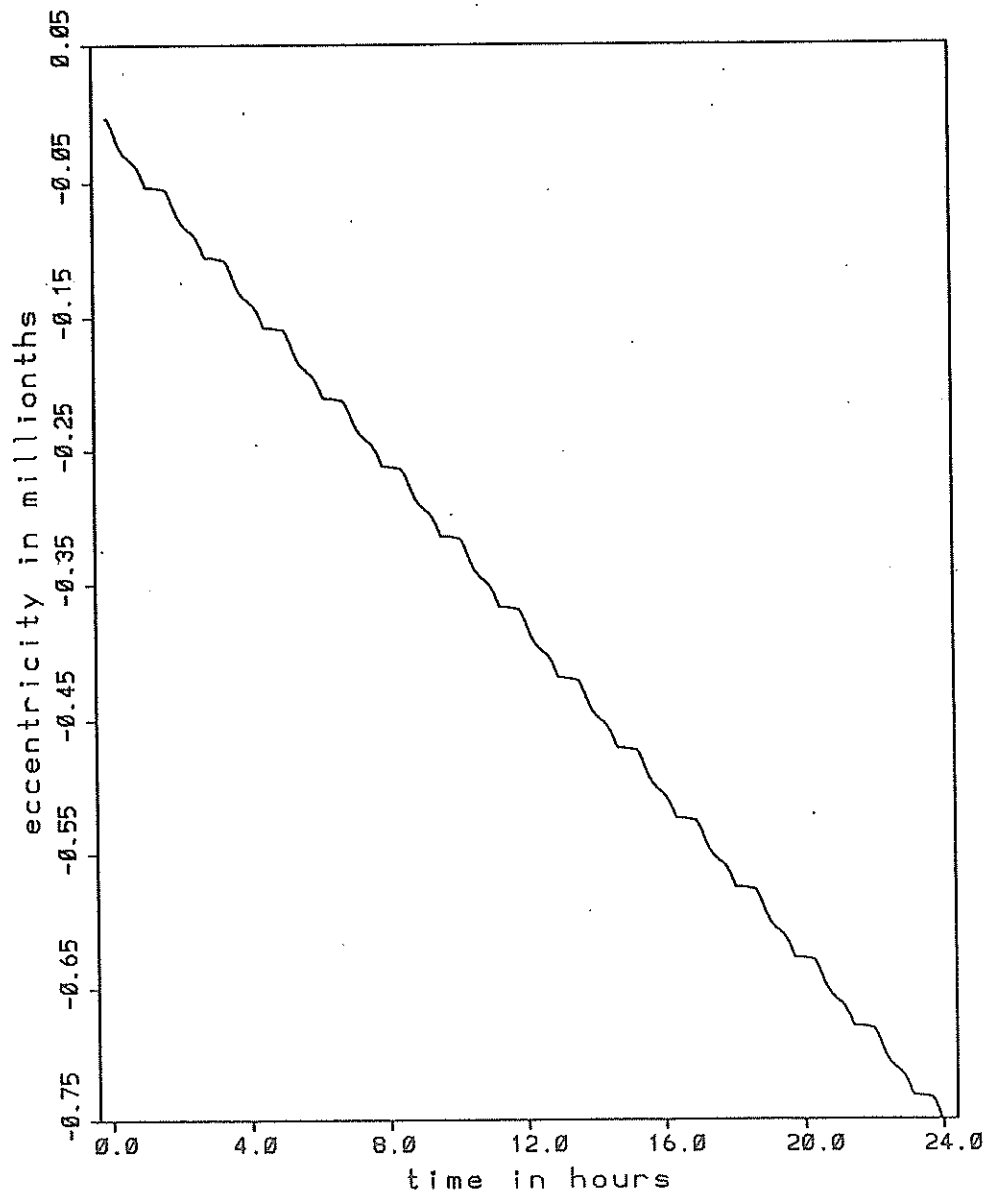
				geomagnetic index Kp			
				Δa (m)	1.0	4.0	7.0
ssof = sof				100	0.40	0.58	0.87
				150	0.97	1.47	2.28
				200	2.25	3.90	6.07

				geomagnetic index Kp		
				Δa (m)	1.0	4.0
sof	150	ssof	100	0.52	0.79	1.17
	200		150	1.31	1.98	3.06
	250		200	3.38	5.18	7.97

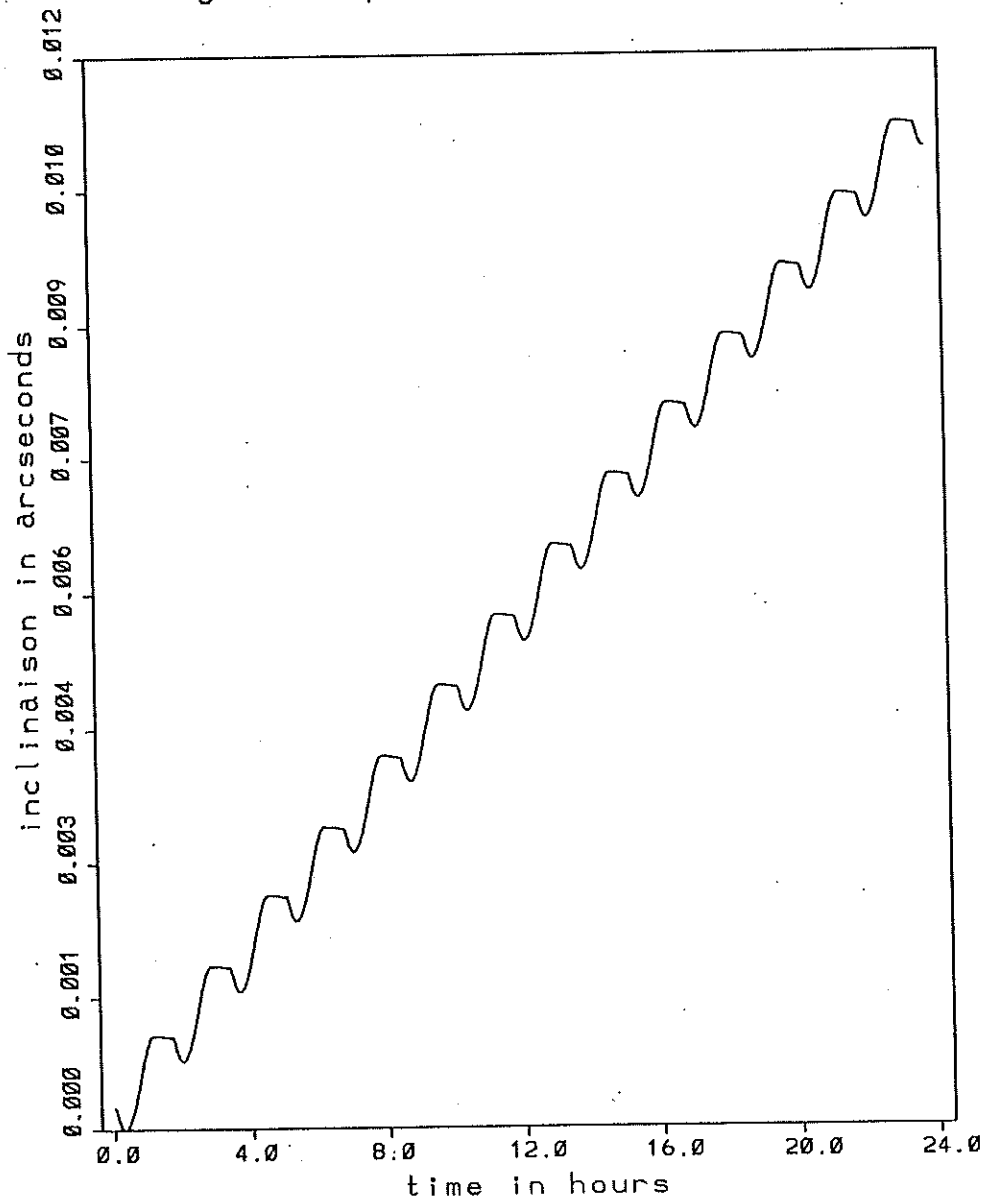
thermal forces influence on the semi-major-axis
during one day-arc of spot on June 23, 1989



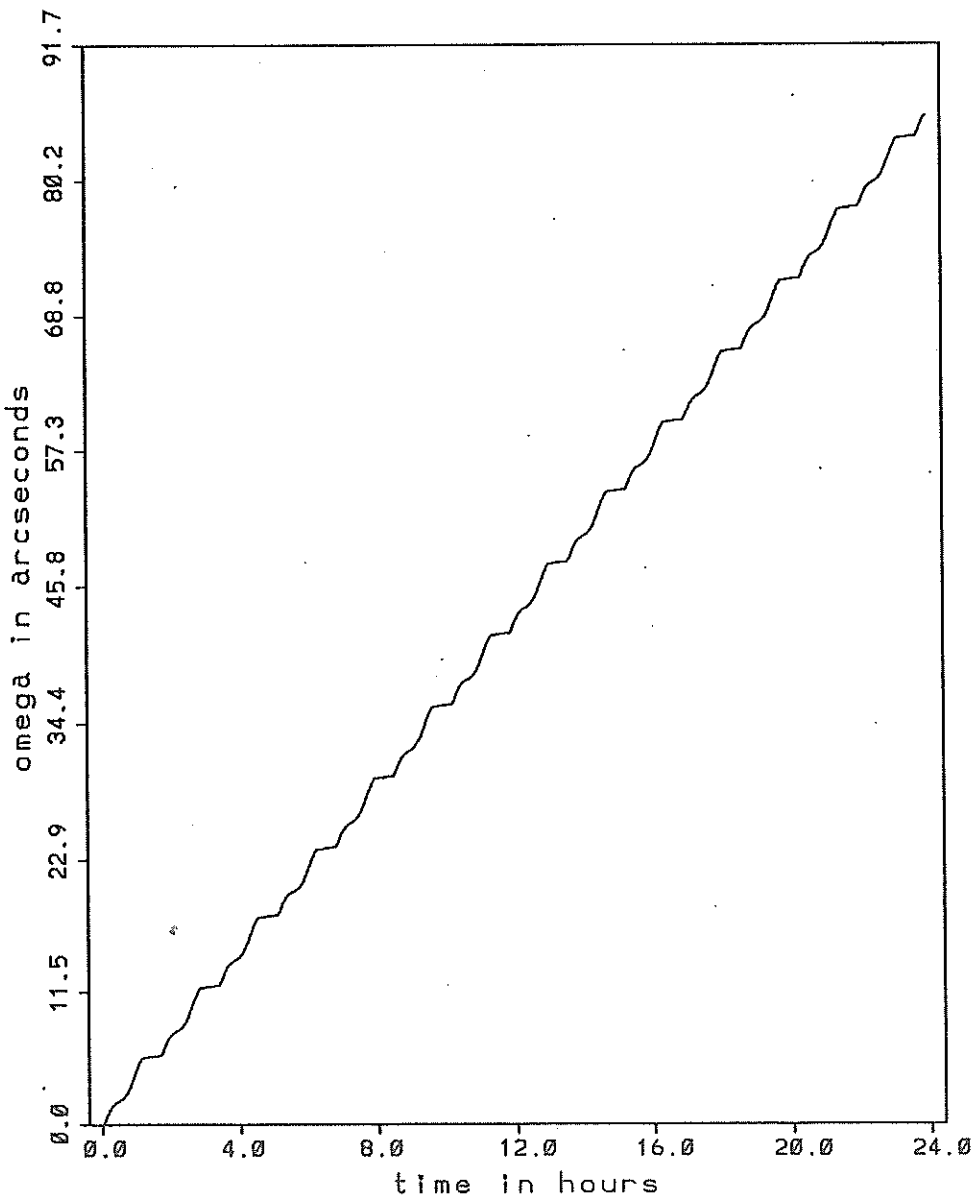
thermal forces influence on the eccentricity
during one day-arc of spot on June 23, 1989



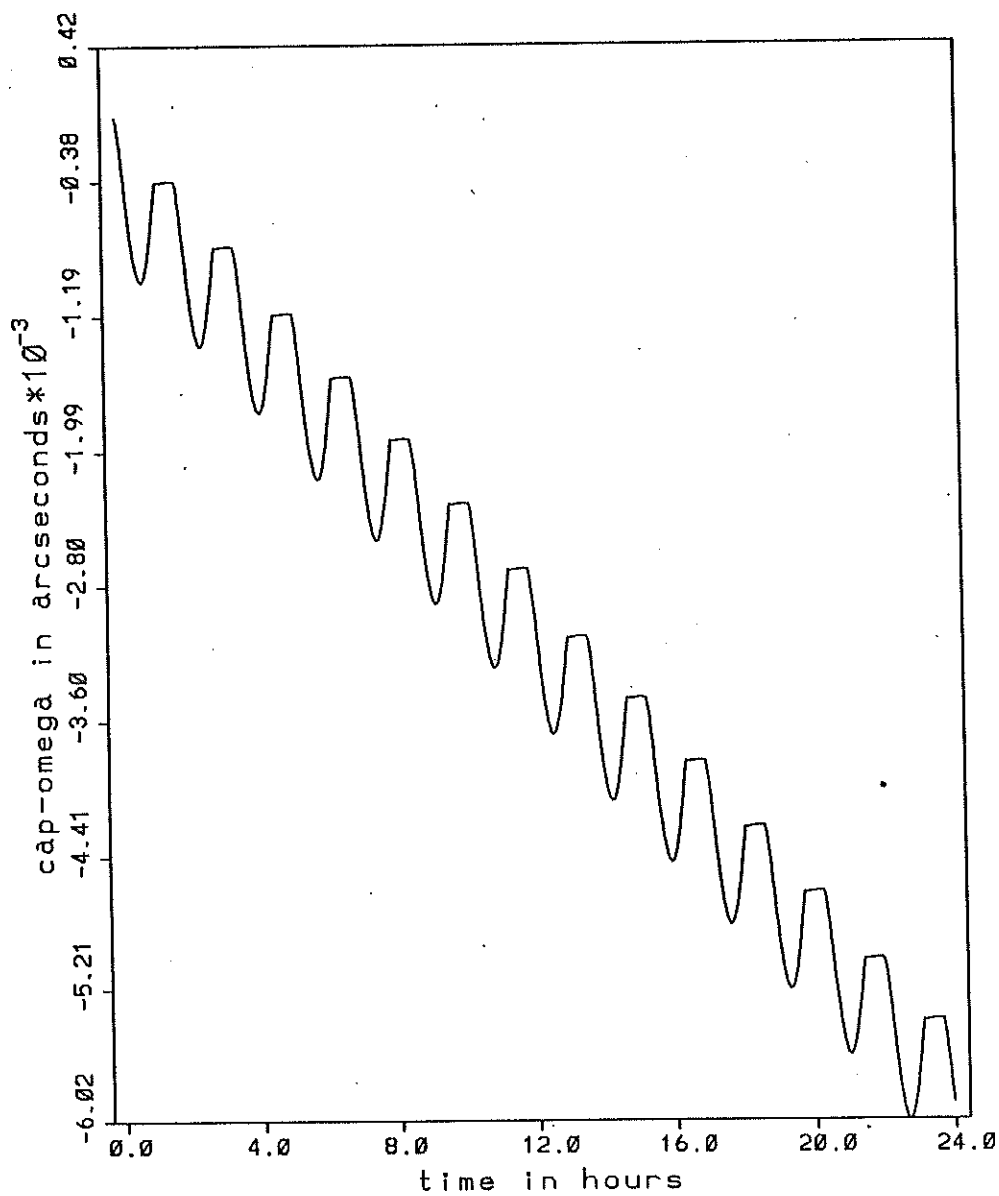
thermal forces influence on inclination
during one day-arc of spot on June 23, 1989



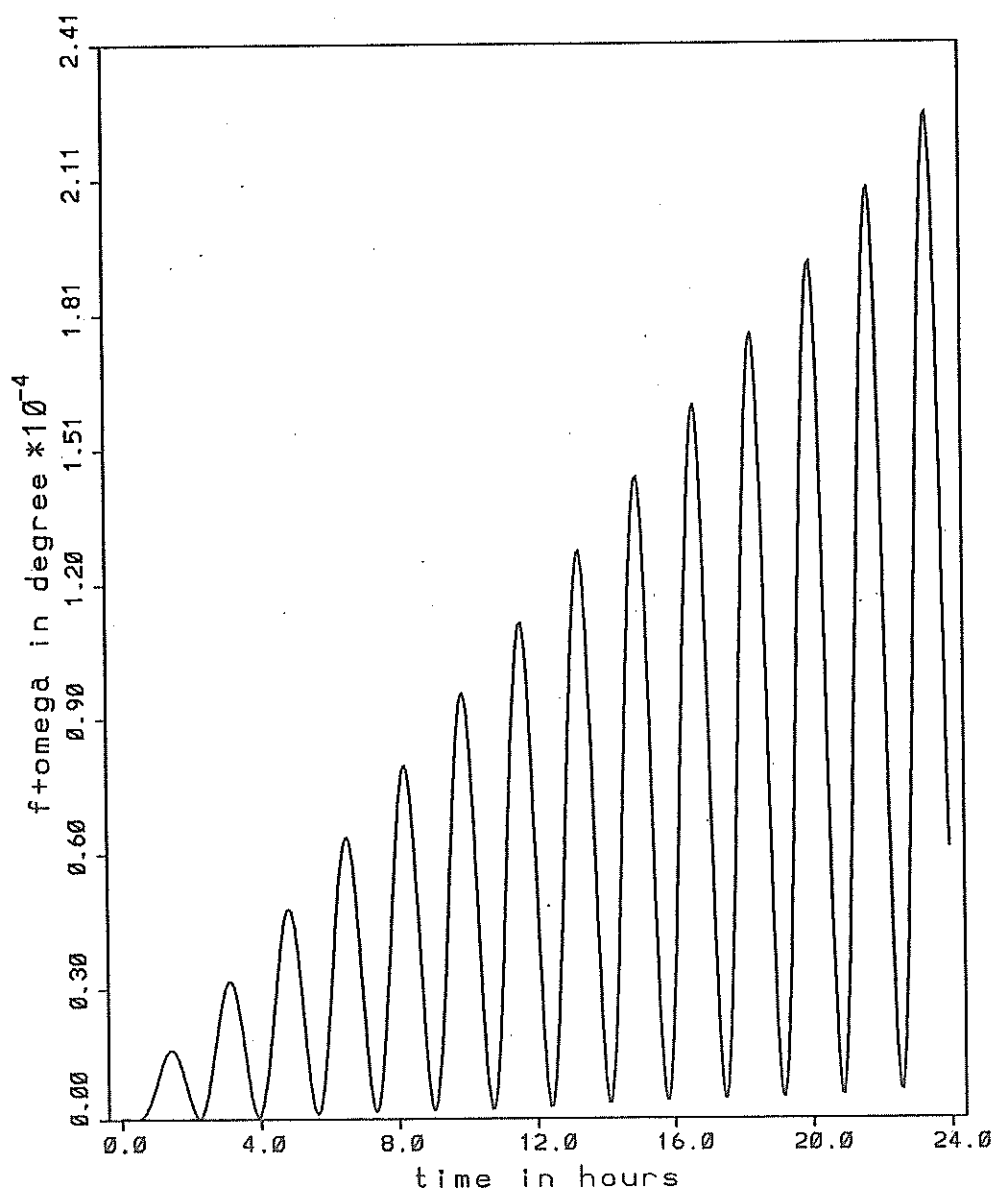
thermal forces influence on omega
during one day-arc of spot on June 23, 1989



thermal forces influence on cap-omega
during one day-arc of spot on June 23, 1989



thermal forces influence on $f+\omega$
during one day-arc



thermal forces influence on the mean anomaly
during one day-arc

