RENEWED INTEREST IN ADJUSTING STOCHASTIC ACCELERATIONS FOR DORIS-ONLY PRECISE ORBIT SOLUTIONS

F. MERCIER, J. MOYARD, A. COUHERT, S. HOURY, S. BLONDEL

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CASE OF CRYOSAT-2

Characteristics:

- Launch: April 8, 2010 => Oldest (among currently flying) altimetry mission
- Altitude: 717 km
- Inclination: polar orbit (92°)
- Tracking instruments: DORIS, LRA (no GNSS receiver)

Typical dynamic orbit solution:

- 7-day orbit arcs
- Thermosphere model rescaled every 3 hours (~2 revolutions) following the updates of the geomagnetic indexes
- Unconstrained along-track 1/rev accelerations estimated every 12h
- Unconstrained single cross-track 1/rev acceleration adjusted over the weekly arc (residual J2 and Solar Radiation Pressure modeling errors)

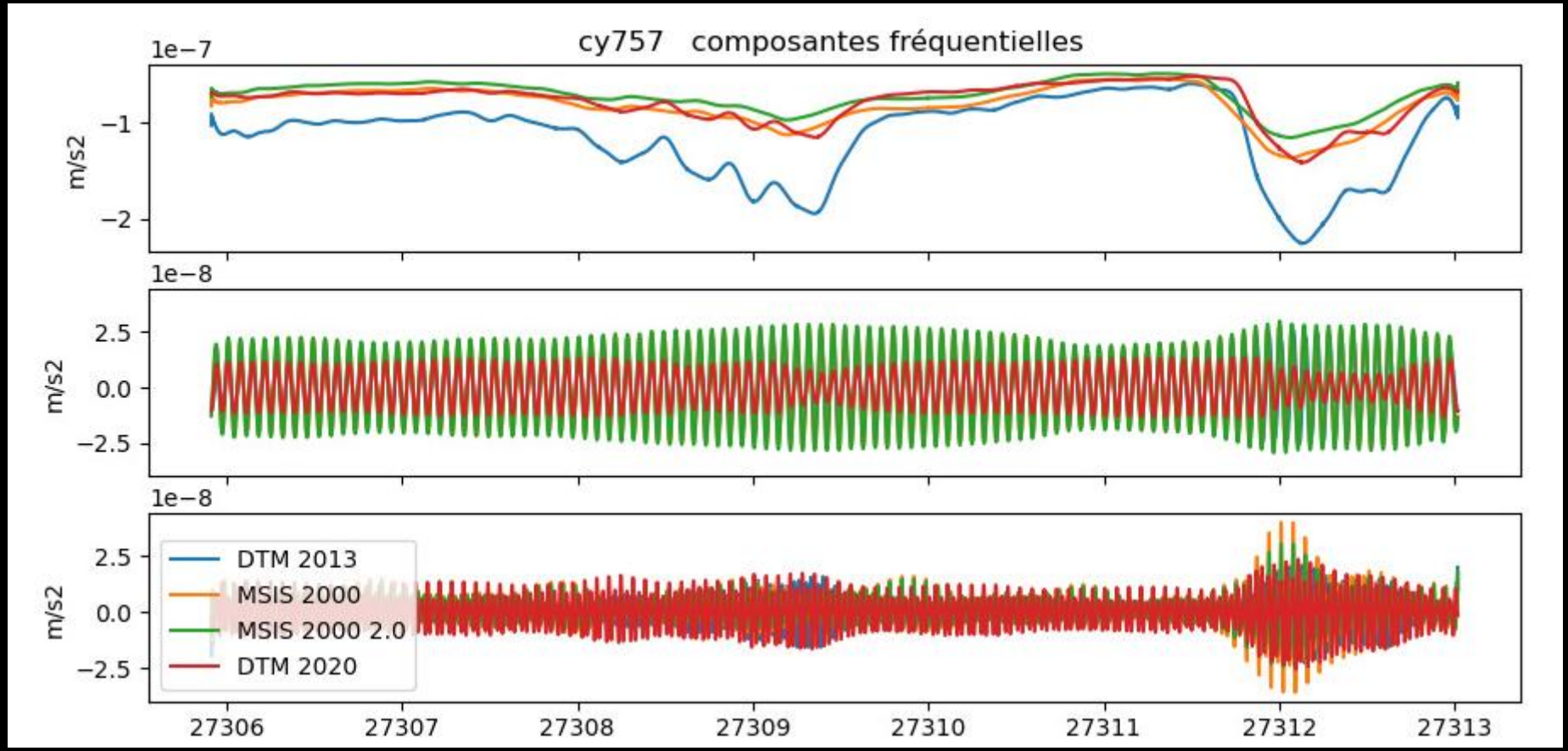
=> Overall good performances, except for high solar activity periods

THERMOSPHERE MODEL INDUCED ALONG-TRACK ACCELERATIONS

Long-term

1/rev

>1/rev

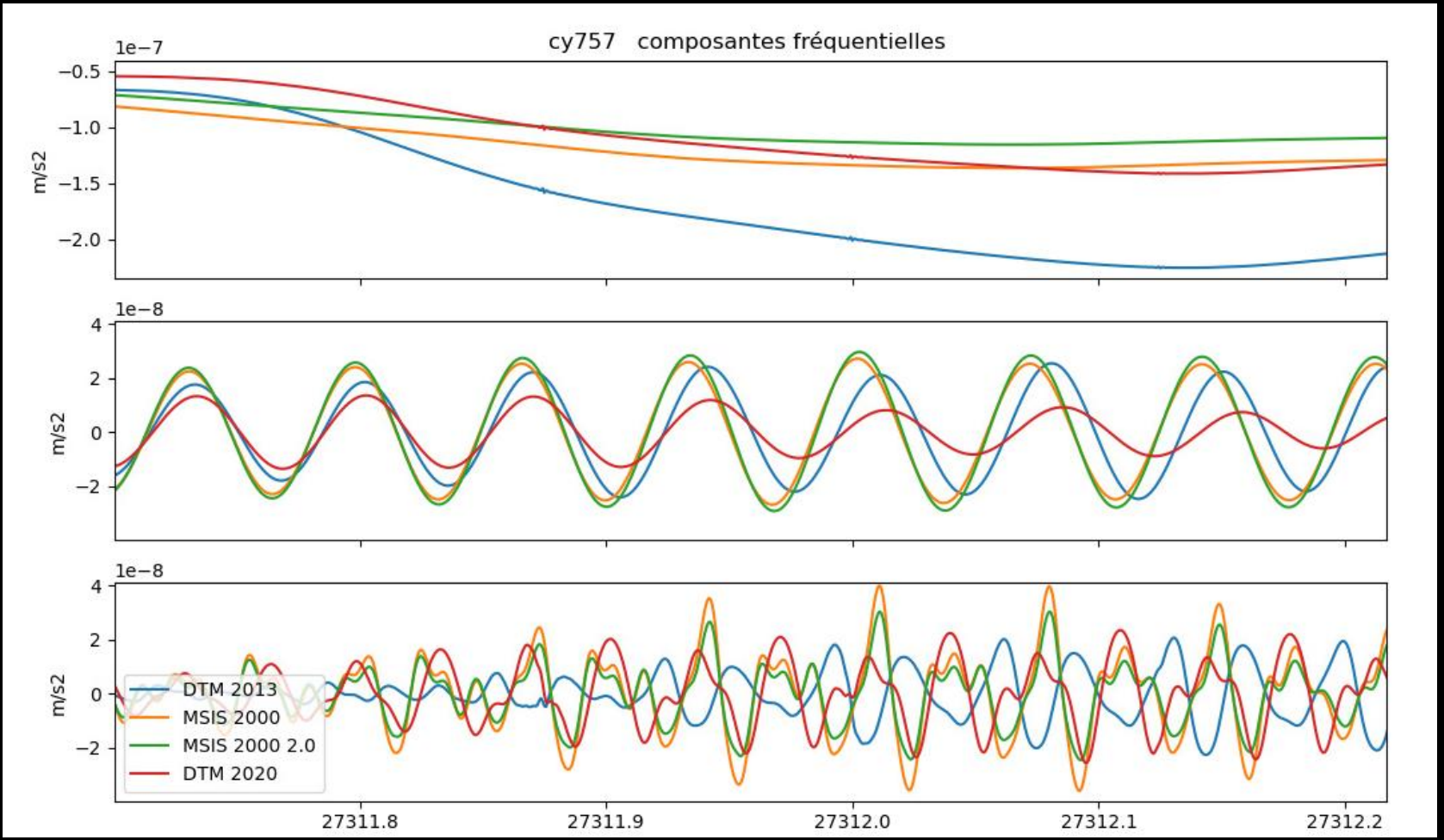


THERMOSPHERE MODEL INDUCED ALONG-TRACK ACCELERATIONS (ZOOM IN)

Long-term

1/rev

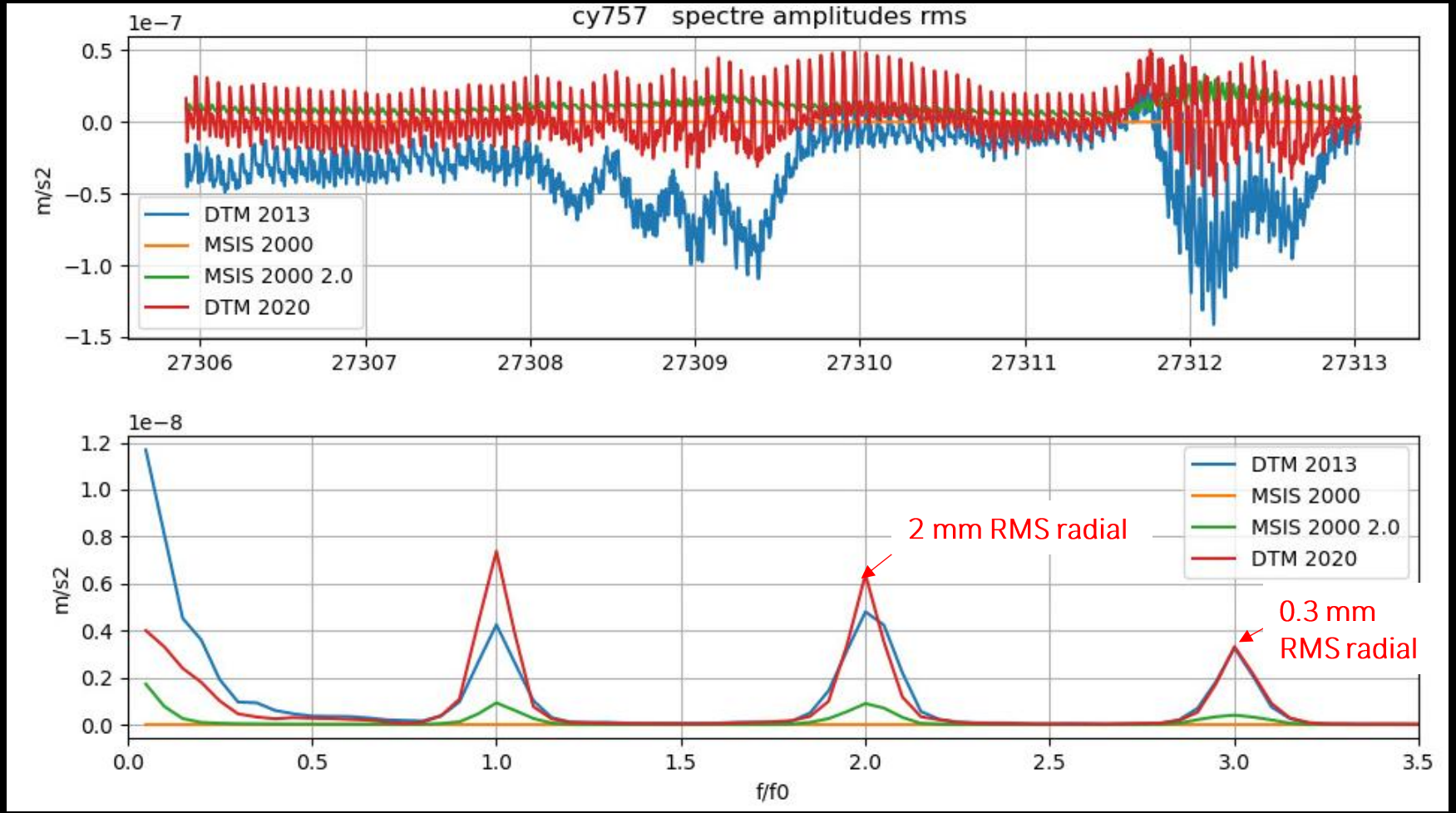
>1/rev



THERMOSPHERE MODEL INDUCED ALONG-TRACK ACCELERATIONS WRT MSIS 2000

Temporal comparisons

Spectrum differences



INTRODUCING STATE PROCESS NOISE FOR UNMODELED ACCELERATIONS

Goal: model the time correlation (over a wide range of frequencies) of un(mis)modeled accelerations (e.g., drag) to improve the orbit accuracy

Approach: augmentation of the state vector with empirical accelerations modeled as first-order Gauss-Markov processes that treat the unmodeled accelerations as exponentially correlated in time

- First step: Standard dynamic orbit to unbias the background models

-Second step: Redetermination of the measurement parameters p with the initial position/velocity x_0 of the spacecraft; while estimating stochastic process noise accelerations x_k with a zero-mean white Gaussian process covariance Q_J, as variations around the previous dynamic orbit which are constant over the time batch Δt, and characterized by the correlation time τ with steady-state variance $\sigma_p{}^2$ (Barotto, 1995)

- $\frac{\Delta t}{\tau} \to 0$: Random walk processes (if σ_p is big enough)
- $\frac{\Delta t}{\tau} \to +\infty$: White noise processes with variance $\sigma_p{}^2$

$$J(p,x) = \sum_{meas.} \frac{(y_m - y(p,x_0))^2}{\sigma_m^2} + \sum_k \frac{(x_{k+1} - e^{-\frac{\Delta t}{\tau}} x_k)^2}{Q_J}$$

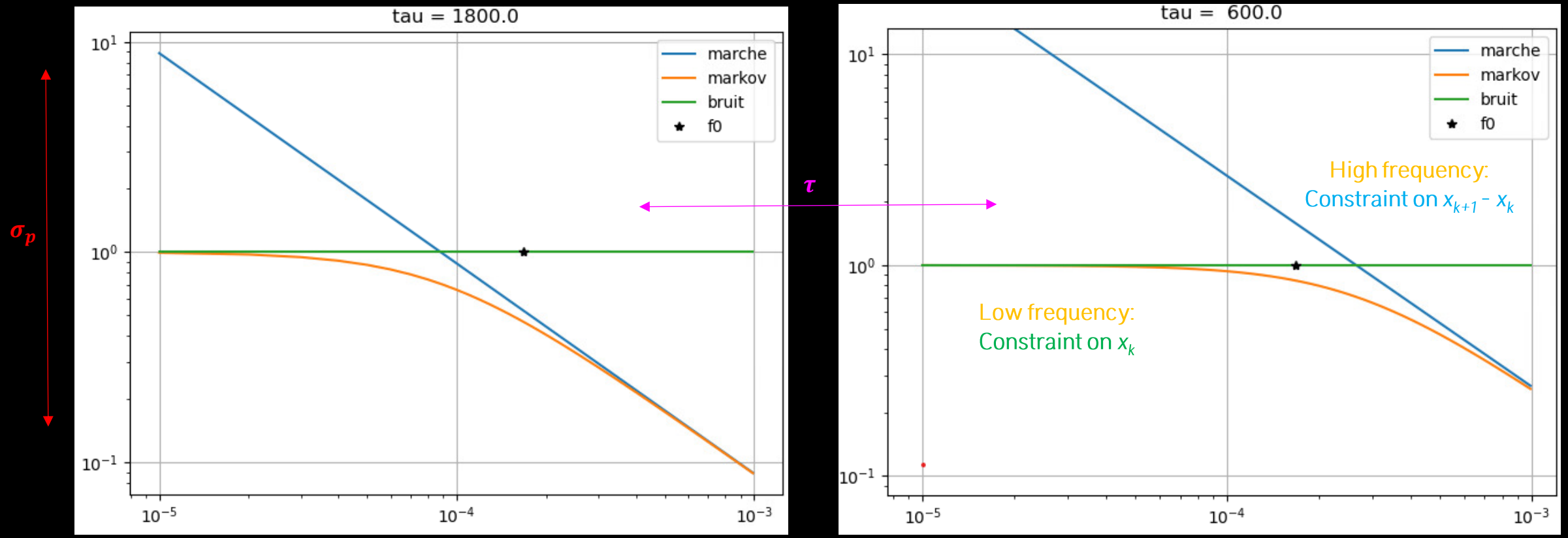
$$Q_J = (1 - e^{-2\frac{\Delta t}{\tau}})\,\sigma_p^2$$

$$\frac{\Delta t}{\tau} \to 0 : Q_J = 2\Delta t \frac{\sigma_p^2}{\tau}$$

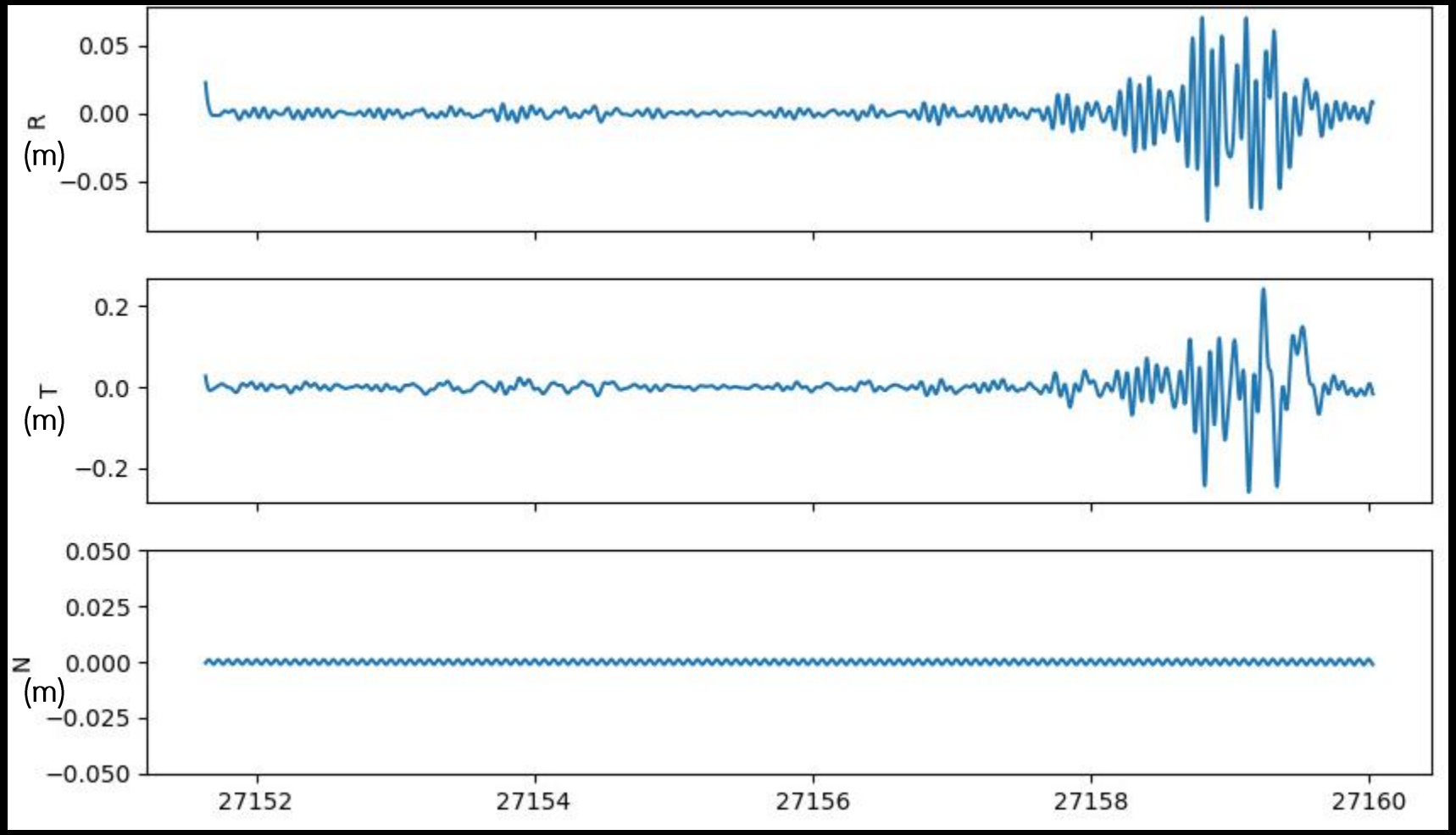
$$\frac{\Delta t}{\tau} \to +\infty : Q_J = \sigma_p^2$$

FREQUENCY REPRESENTATION OF A FIRST-ORDER GAUSS-MARKOV PROCESS

How to choose τ and σ_p?

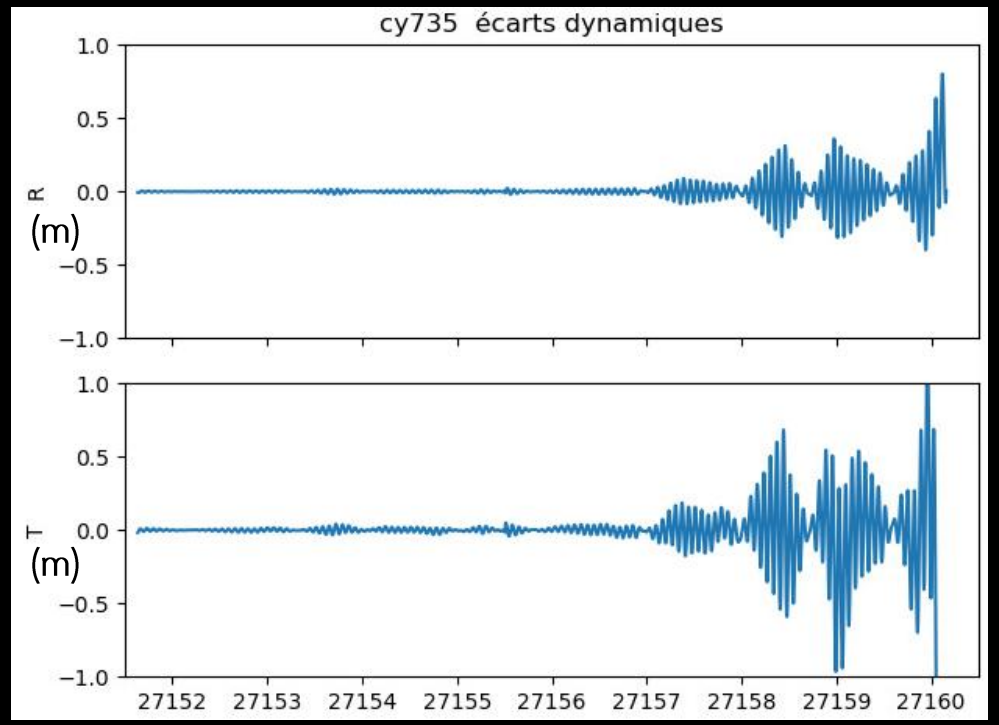


ORBIT CORRECTIONS PERFORMED BY ALONG-TRACK STOCHASTIC ACCELERATIONS

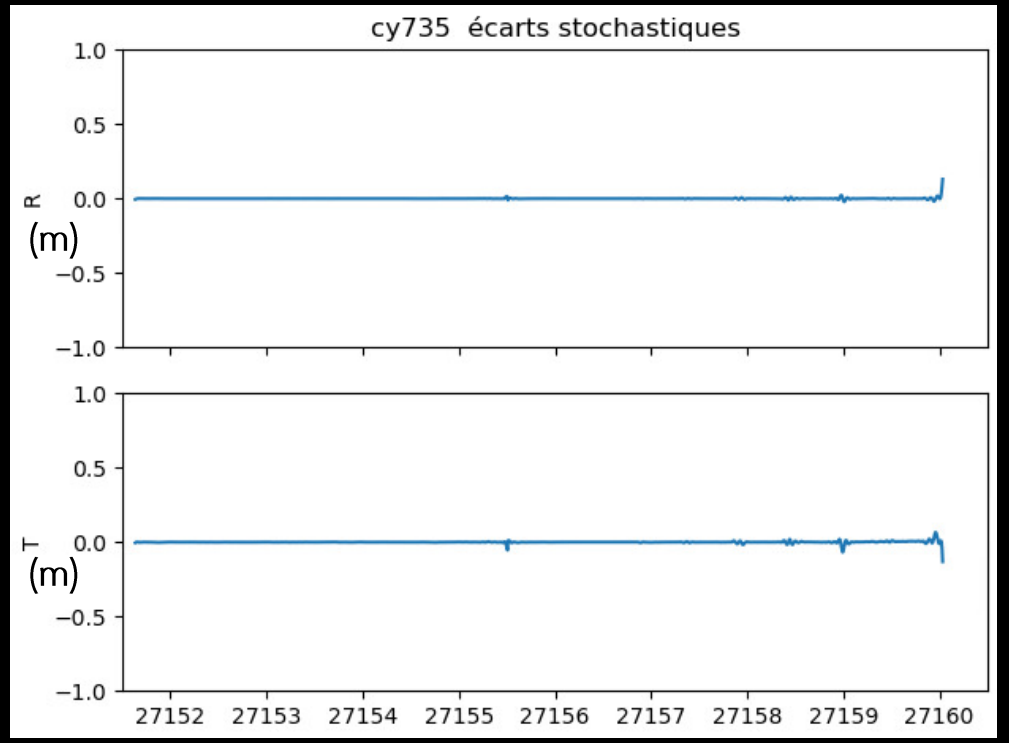


ORBIT CORRECTIONS PERFORMED BY ALONG-TRACK STOCHASTIC ACCELERATIONS

Differences between two dynamic orbits with daily along-track 1/rev empirical accelerations shifted by 12h: **Before the stochastic corrections (left)**



After the stochastic corrections (right)

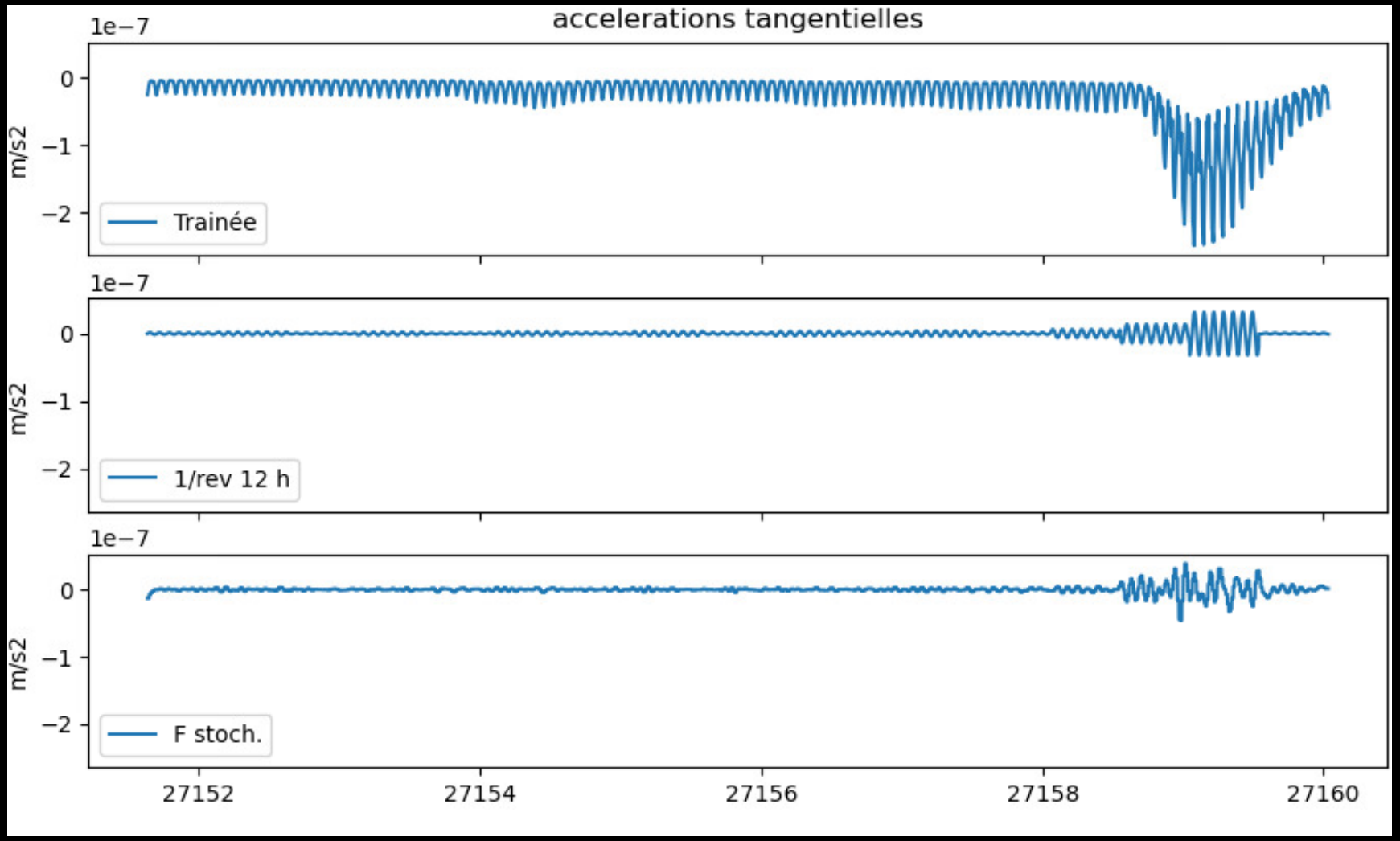


ALONG-TRACK ACCELERATION COMPARISONS

Drag

12h-empircal 1/rev (dynamic)

Stochastic

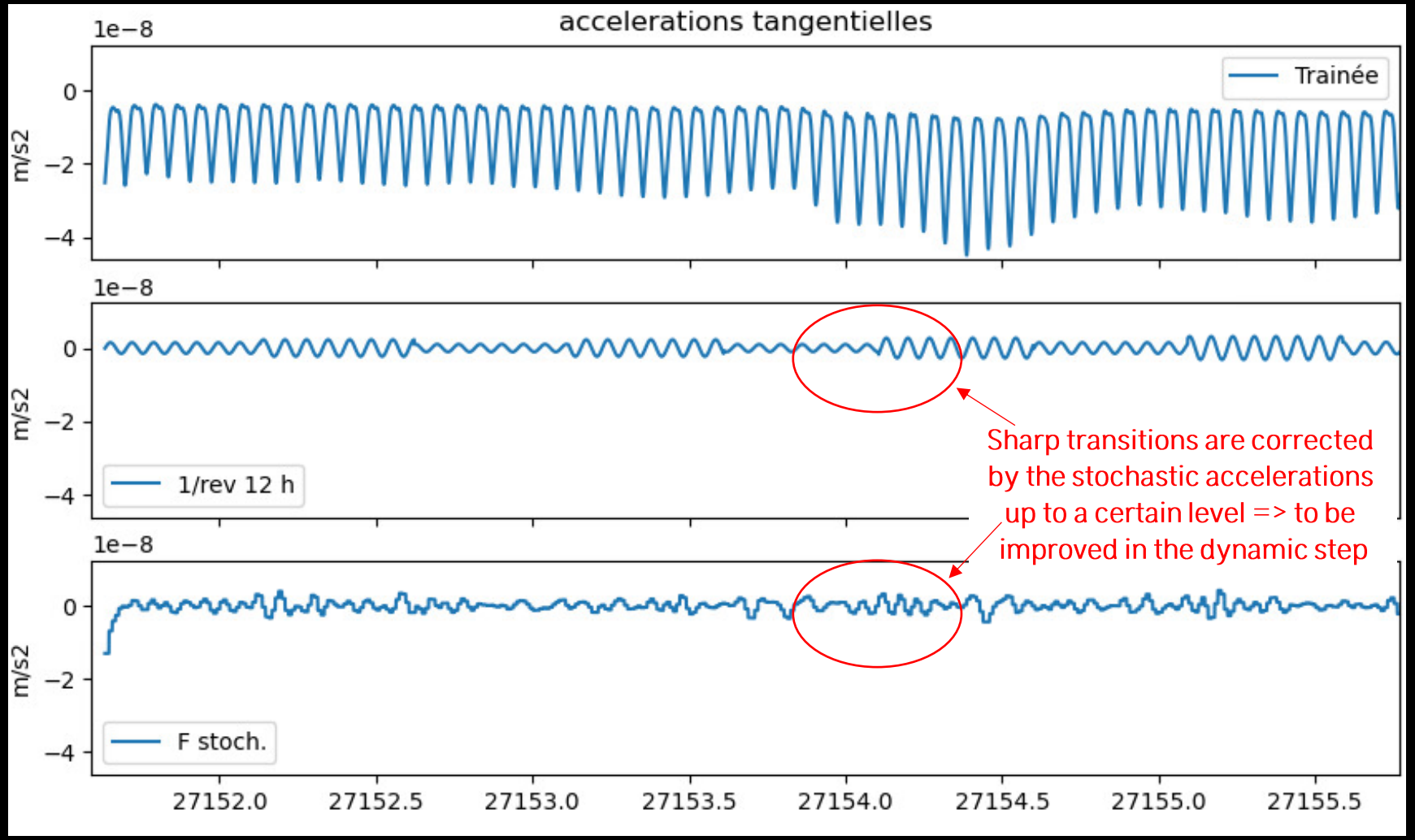


ALONG-TRACK ACCELERATION COMPARISONS (ZOOM IN)

Drag

12h-empircal 1/rev (dynamic)

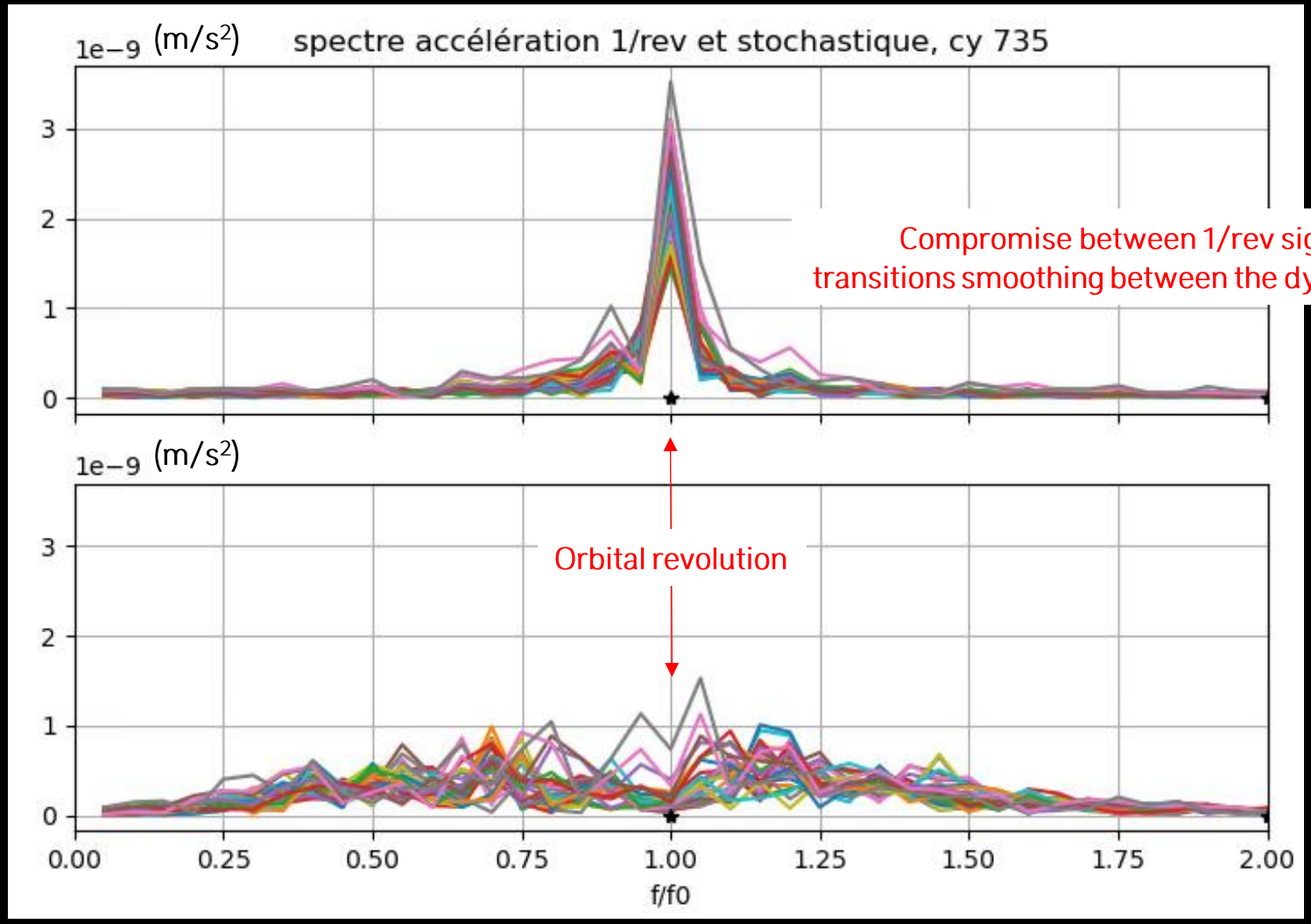
Stochastic



DYNAMIC VS STOCHASTIC ACCELERATION SPECTRUMS

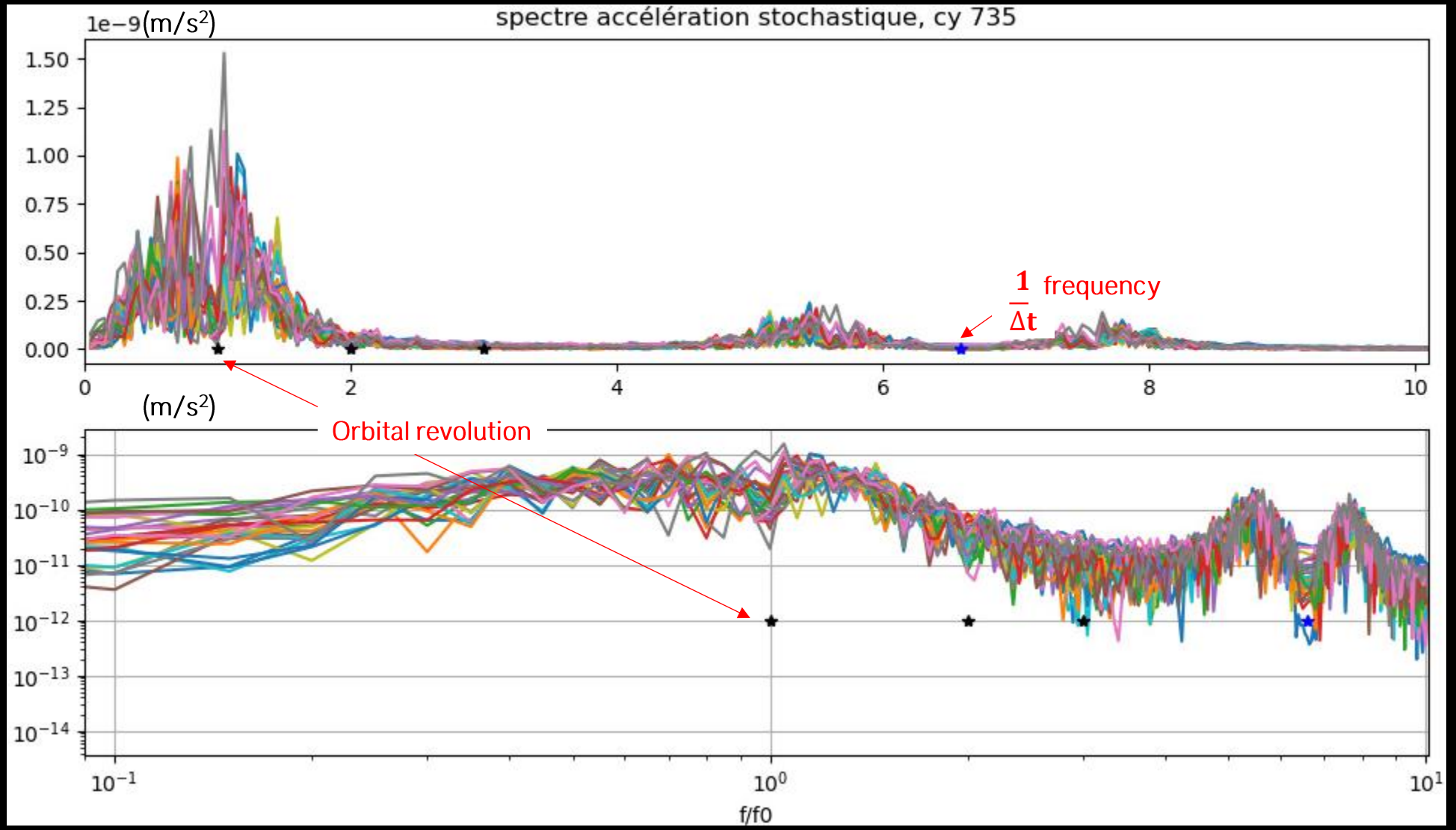
12h-empircal 1/rev (dynamic)

Stochastic



Compromise between 1/rev signals elimination and transitions smoothing between the dynamic empirical segments

STOCHASTIC ACCELERATION SPECTRUM

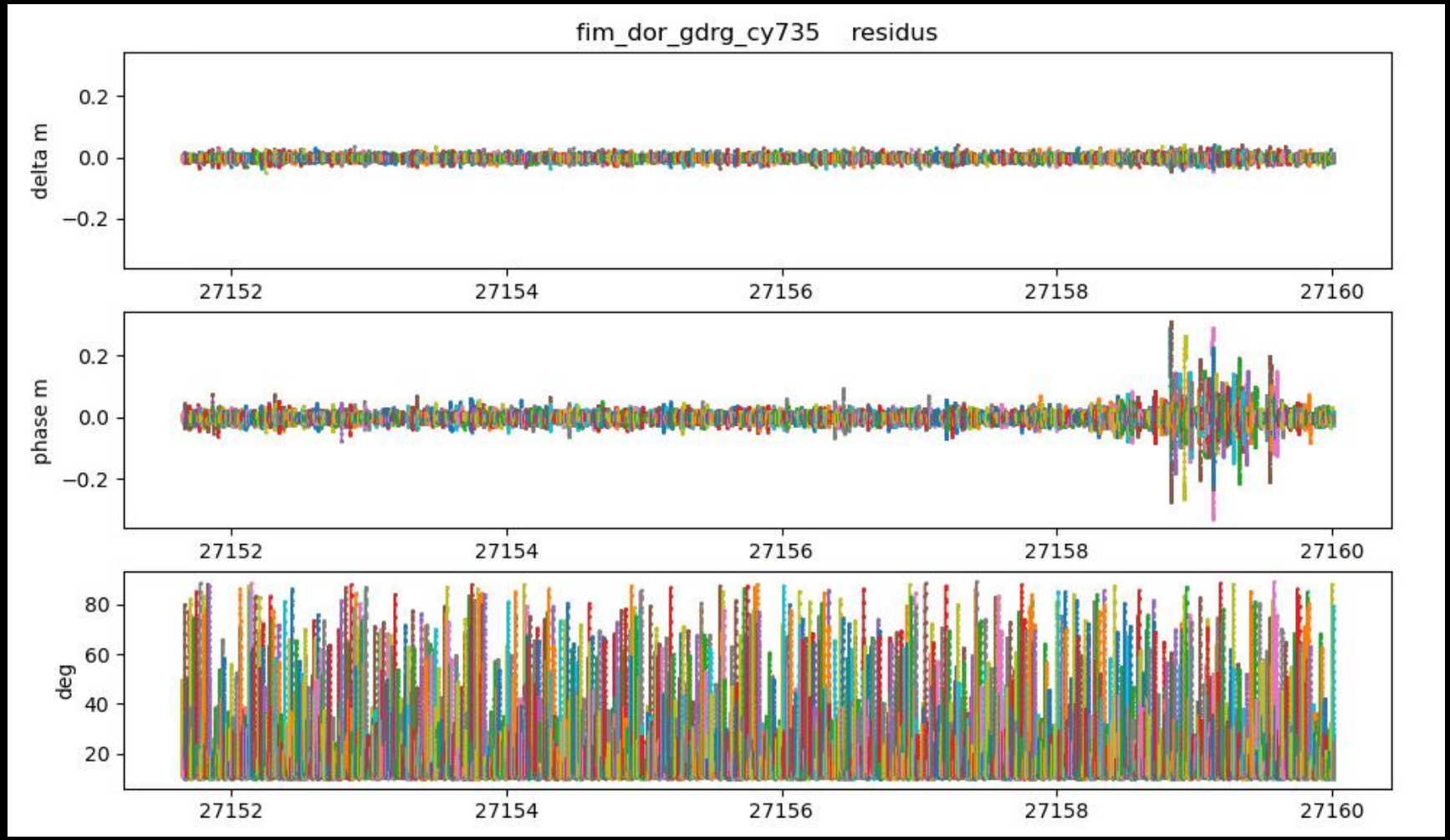


ALONG-TRACK MODELING ERROR SIGNATURE IN THE PHASE RESIDUALS (DYNAMIC)

Doppler residuals

Phase residuals

Elevation

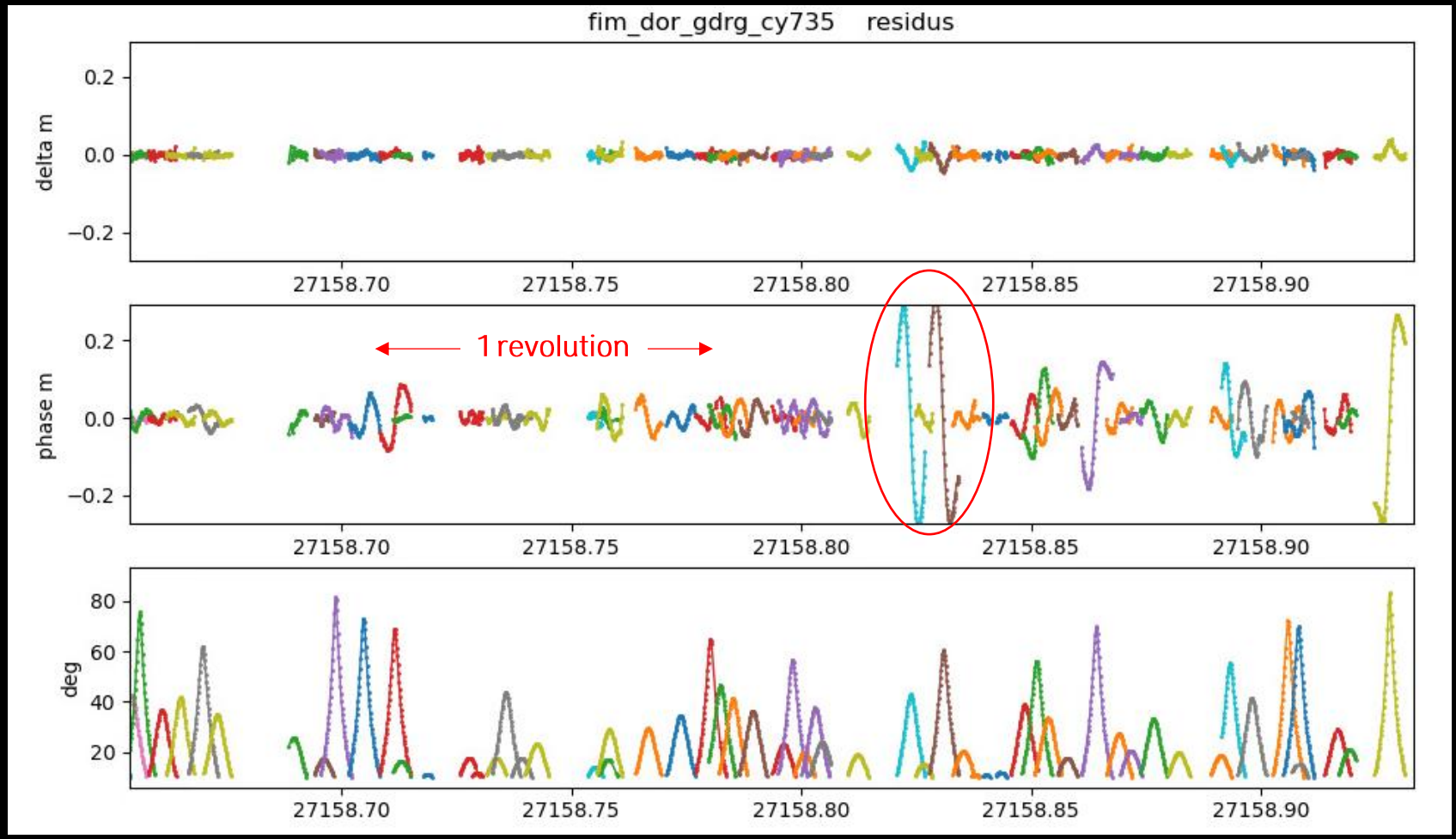


ALONG-TRACK MODELING ERROR SIGNATURE IN THE PHASE RESIDUALS (DYNAMIC)

Doppler residuals

Phase residuals

Elevation

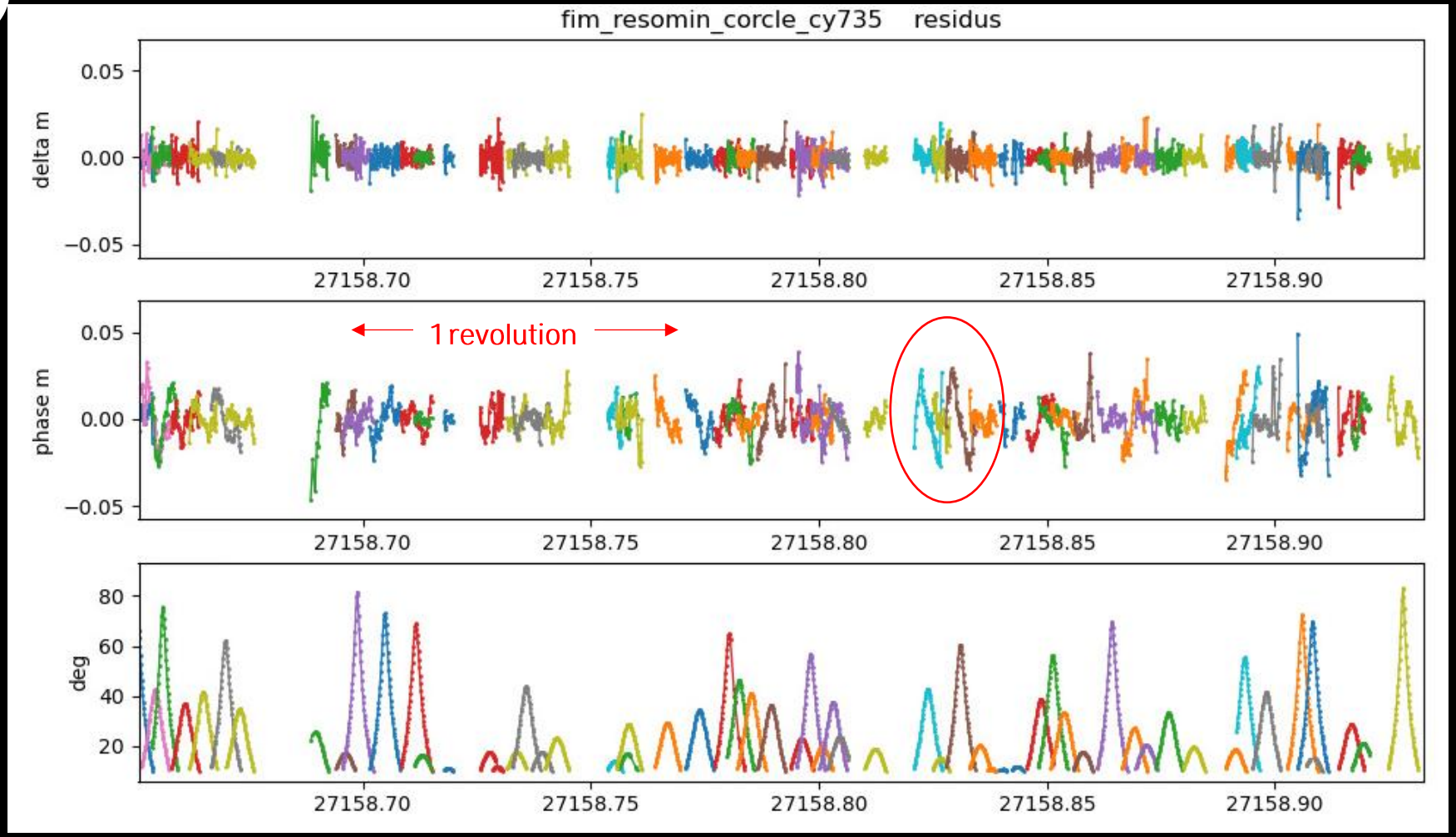


ALONG-TRACK MODELING ERROR SIGNATURE IN THE PHASE RESIDUALS (STOCHASTIC)

Doppler residuals

Phase residuals

Elevation



LESSONS LEARNED FROM THE CRYOSAT-2 STUDY

PRELIMINARY RESULTS FOR SARAL/ALTIKA

Optimal parameterization: with an assumed noise of 5 mm for DORIS measurements

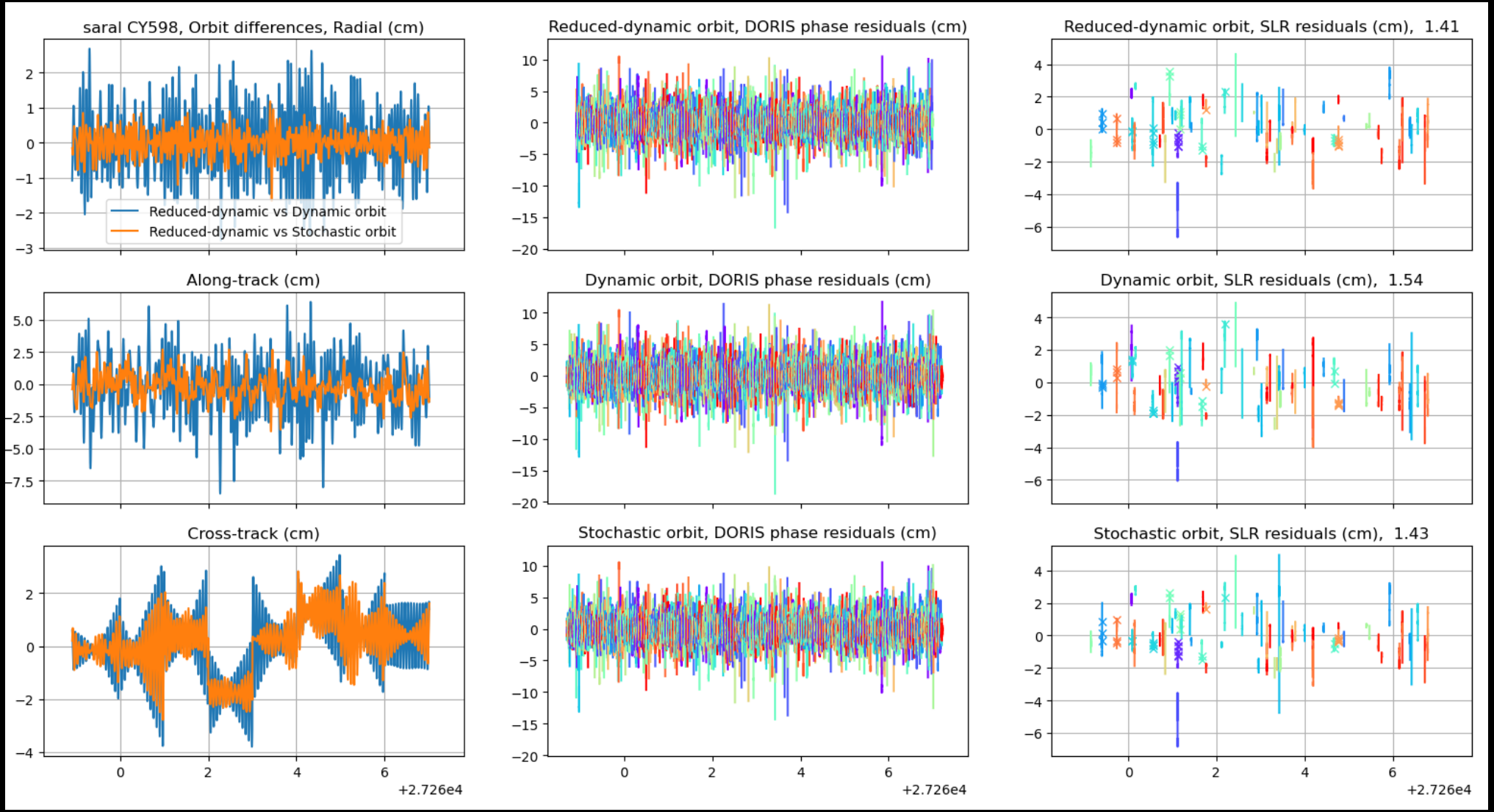
- Dynamic orbit:
  - Thermosphere model rescaled every 3 hours (~2 revolutions) following the updates of the geomagnetic indexes
  - Along-track 1/rev accelerations estimated every 12h **with relative constraints (10^{-10} m/s^2)** => Cope with abrupt twice-daily transitions
  - **Daily** cross-track 1/rev and **constant** accelerations **with relative constraints (10^{-9} m/s^2)** => Mitigate daily J2 and SRP variations while smoothing transitions between the daily segments

- Stochastic solution: *(first-order Gauss-Markov processes)*
  - Batch length = 60 s
  - Correlation time = 300 s
  - Steady state variance:
    - Radial: no stochastic radial acceleration adjusted
    - Along-track: 10^{-8} m/s^2
    - Cross-track: 10^{-8} m/s^2

- Reduced dynamic orbit:
  - 30-min constant accelerations with relative constraints of 5 10^{-9} m/s^2 *(random walk process)*
  - 1/rev along-track and cross-track accelerations estimated every orbital revolution with relative constraints of 10^{-9} m/s^2 and 5 10^{-10} m/s^2, respectively *(random walk processes)*
  - Unconstrained daily constant cross-track accelerations *(white noise process)*

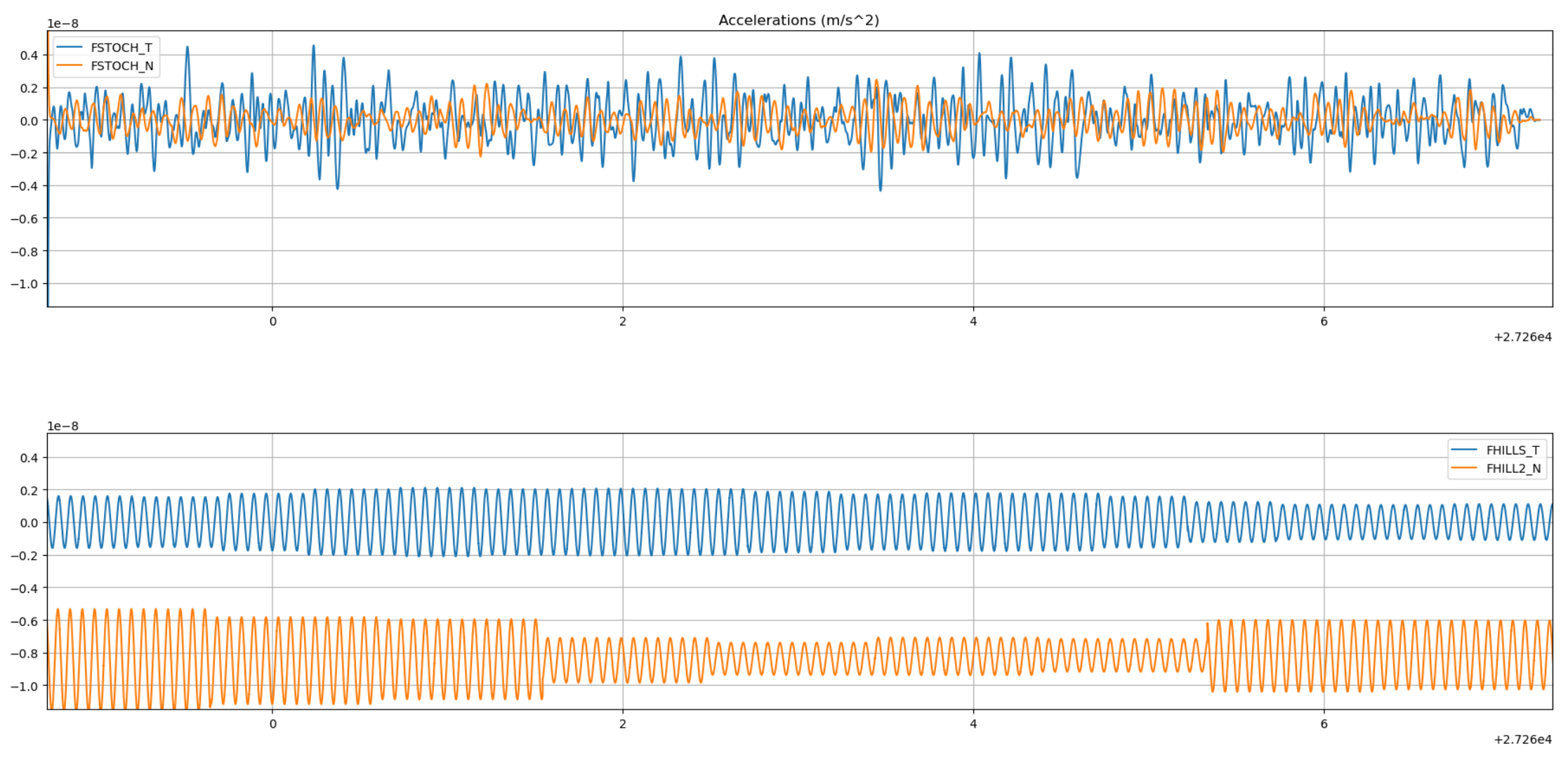
ORBIT QUALITY IN LOW SOLAR ACTIVITY

PRELIMINARY RESULTS
FOR SARAL/ALTIKA



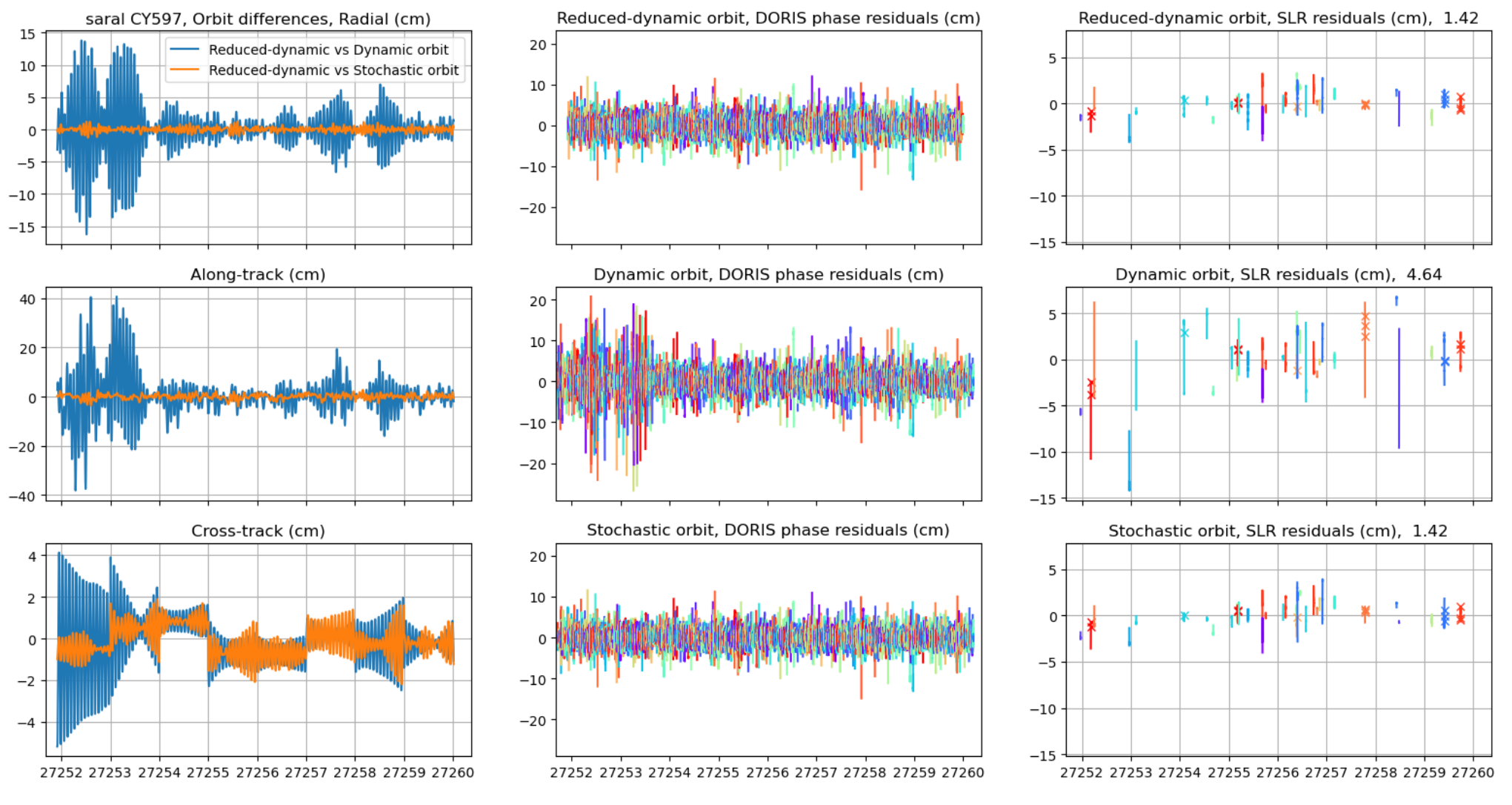
STOCHASTIC VS DYNAMIC ACCELERATIONS

PRELIMINARY RESULTS FOR SARAL/ALTIKA



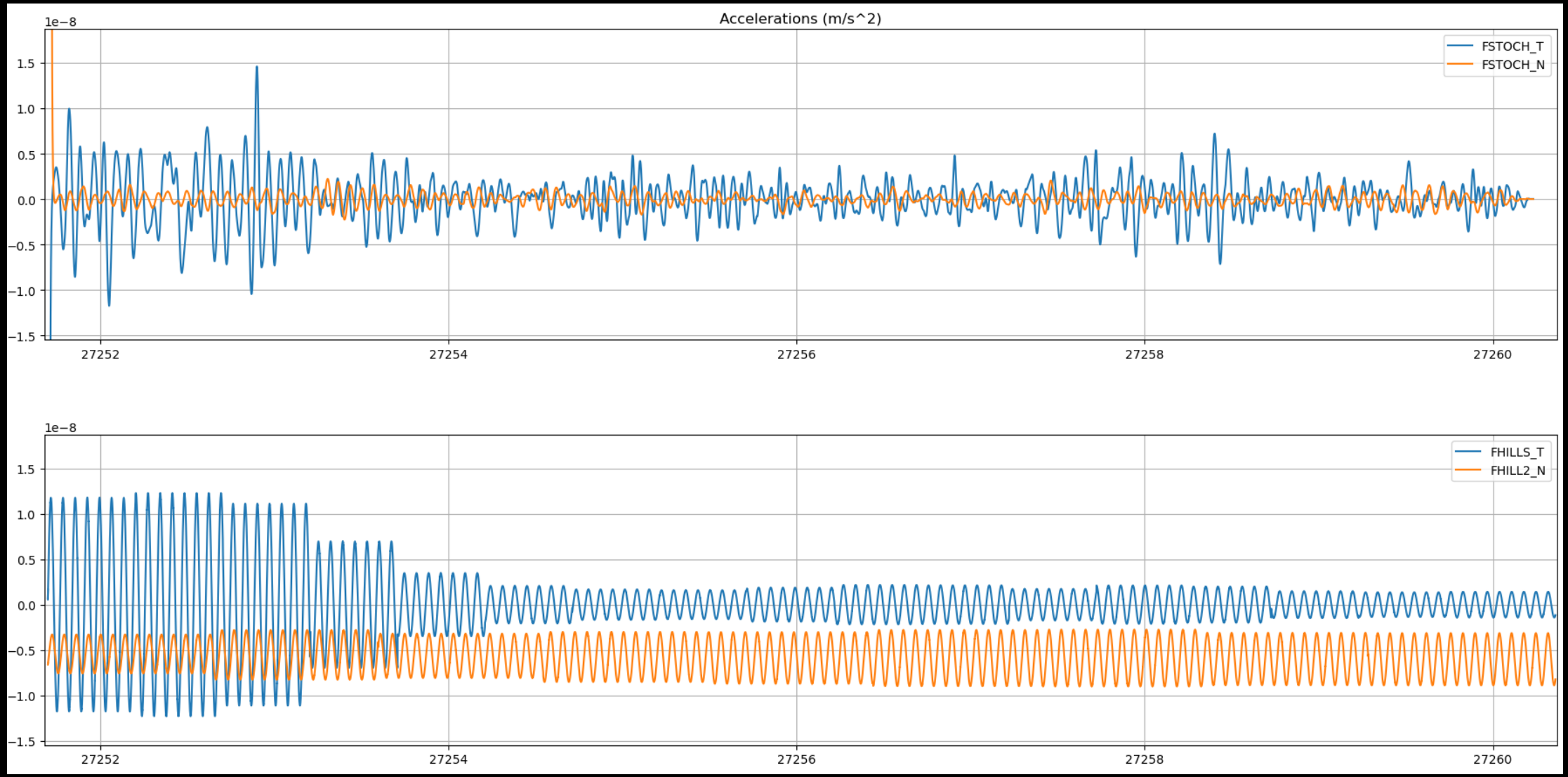
ORBIT QUALITY IN HIGH SOLAR ACTIVITY

PRELIMINARY RESULTS
FOR SARAL/ALTIKA



STOCHASTIC VS DYNAMIC ACCELERATIONS

PRELIMINARY RESULTS FOR SARAL/ALTIKA



AMONG THE STARS ON THE HOLLYWOOD WALK OF FAME